
An Example of Macroeconomic Theory Comprehensive Examination

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- Consider the real business cycle (RBC) model with GHH preferences, which is described below.
 - There is the representative household whose preferences over consumption (c_t) and labor supply (n_t) are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \quad (0.1)$$

where the functional form of the period utility is given as the GHH (Greenwood, Hercowitz and Hoffman, 1988) preference specification:

$$U(c_t, n_t) = \log \left(c_t - \theta \frac{n_t^{1+\chi}}{1+\chi} \right). \quad (0.2)$$

The household owns capital stock (k_t) and rents it to firms at the real rental rate of capital (r_t^k). It also holds risk-free real bonds (b_t) which pay the real interest rate (r_t) at time t . Because the household owns firms and receive their profits (Π_t), its period budget constraint can be expressed as:

$$c_t + I_t + b_{t+1} = w_t n_t + r_t^k k_t + (1 + r_t) b_t + \Pi_t, \quad (0.3)$$

where I_t is investment and w_t is real wages. The law of motion on capital accumulation is:

$$k_{t+1} = I_t + (1 - \delta) k_t. \quad (0.4)$$

- There is also a representative firm. It produces goods y_t , which are used as consumption or investment, using labor and capital, according to the following production function:

$$y_t = a_t k_t^\alpha n_t^{1-\alpha}. \quad (0.5)$$

- The process of total factor productivity (a_t) is exogenously given by:

$$\log(a_t/a^*) = \rho_a \log(a_{t-1}/a^*) + \varepsilon_{at}, \quad (0.6)$$

where a^* is the steady state value of TFP and assumed to be one (i.e., $a^* = 1$).

- The aggregate feasibility (or goods market clearing) condition is:

$$y_t = c_t + I_t. \quad (0.7)$$

- Answer the following questions.

1. Construct a current value Lagrangian for the household problem and then find the first-order conditions with respect to c_t , n_t , k_{t+1} and b_{t+1} . In particular, log-linearize the labor supply condition (i.e., the first-order condition with respect to n_t) using notations for variables with tilde that represent percentage deviations from their steady state values, and then make an economic interpretation of the parameter χ in the GHH preference specification.
2. Write down the firm problem and then find the first-order conditions with respect to labor and capital. Note that since we assume that the firm does not issue debts and the household owns capital stock, the firm problem is static. In particular, log-linearize the labor demand condition (i.e., the first-order condition with respect to labor).
3. Write down the system of equations characterizing 10 equilibrium variables including λ_t :

$$(c_t, n_t, y_t, k_{t+1}, I_t, w_t, r_t^k, r_{t+1}, a_t, \lambda_t)$$

where λ_t is the Lagrangian multiplier associated with the household problem. Then, calculate the steady state values of all variables in terms of model parameters. Note that the steady state of a_t is assumed to be one (i.e., $a^* = 1$).

4. Suppose the economy is initially at its steady state. At time 0, there is a positive shock to TFP so that \tilde{a}_0 (i.e., percentage deviations of TFP from its steady state value) increases by one percentage point.
 - (a) Analyze its effects on labor (\tilde{n}_0) and wages (\tilde{w}_0) on impact of the shock by using log-linearized labor supply and demand equations that you already derived in Questions (1) and (2).
 - (b) Furthermore, compare them for small and large values of the parameter χ in the GHH preference specification, for example, $\chi = 1$ versus $\chi = 2$.