A RATIONAL MODEL OF BRAND

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ABSTRACT. Brand is analyzed as an equilibrium outcome of a game subject to some form of frictions. Brand is a long run capital that generates the flow of products and the associated marketing service. We use an economy with informational friction as a laboratory, to further demonstrate what an equilibrium model of a brand can do, beyond the existing models of brand. In particular, we demonstrate that it can be an optimal for a firm to pool different products, instead of signaling the quality of the individual product in the same tier, in contrast to Milgrom and Roberts (1986). Moreover, the product quality in different tiers can overlaps in an equilibrium brand, contrary to Aribarg and Arora (2008).

KEY WORDS: brand, friction, marketing, asymmetric information, signaling, tier, advertisement

1. INTRODUCTION

Few economic institutions are as pervasive as brand. Yet, there is no consensus in economics, marketing, law, accounting and consumer psychology about what brand is. American Marketing Association defines a brand (Keller (2013)) as "name, term, design, symbol or any other feature that identifies one seller's good or service as distinct from those of other sellers." Still, de Chernatony and Riley (1998) listed 12 definitions of brand: legal instrument, logo, company, shorthand, risk reducer, identity system, image, value system, personality, relationship, adding value, and evolving entity.

The research objective of this paper is to examine rigorously the microeconomic foundation of brand through a game theoretic model, identifying observable implications of an equilibrium outcome. Our first task is to propose a formal definition of brand, which is general enough to be consistent with existing definitions of brand, but also is precise enough to be used for an equilibrium analysis. We investigate an equilibrium model of brand, to understand how various institutional and informational parameters influence the characteristics of a brand, and to derive observable implications, which are useful for structural analysis. We show that our equilibrium analysis offers a fresh new insight toward the inner workings of a brand, beyond what conventional approaches provide.

The production activities can be decomposed into two parts. The first part is the activity that transforms inputs into a good. This part of the production activity is the production in traditional sense. The second part is the activity by the firm, after the good is produced (in traditional sense), but until the good is delivered to a consumer. We regard the second part of the production activities as marketing.

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We define brand as the *long run* capital that generates *both* the flow of marketing service *and* the product over time. In a certain sense, our definition regards brand as the whole company, which is indeed one of commonly used definitions of brand (de Chernatony and Riley (1998)). Our notion of a brand is in line with Bagwell (1989), Simon and Sullivan (1993) and Goldfarb, Lu, and Moorthy (2009), in the sense that we treat brand as an asset to generate excess profit, but differs from the existing definitions in three important ways.

First, *complementarity* between the product and the marketing service associated with the product is the key. Since marketing service is tailored for a particular product, we must consider the product and its marketing service together. Because of complementaries between the product and the marketing service, a brand value is typically larger than the sum of the two components. AT&T wants to buy the brand of Warner Brothers. In this case, the brand includes the library of movie titles owned by Warner Brothers (product asset), but also the distribution network (marketing asset). Without distribution network, the movie library has no value, and vice versa.

Second, a brand is the *long* term capital. Marketing service is designed for a given set of product lines in the short run. However, in the *long* run, the product lines must be optimized as well, in response to various constraints of marketing services. While our notion is closely related to Bagwell (1989), our definition of brand significantly differs from Bagwell (1989) in the same way as the short term capital differs from the long term capital.

Third, we specifically consider *friction* in the economy, as the reason for existence of the marketing asset. In a frictionless economy, the price of a product is determined at its marginal production cost, and the social value of the marketing is 0. In the commodity market, all relevant characteristics of the good are public information. We observe no advertisement. Brand becomes the present discounted value of the product. In the presence of friction such as asymmetric information, however, the price of a product may not be equal to its marginal cost of production, because of lemon's problem. The seller needs the second component of production activity to signal the quality of the product, which now carries social value by increasing the surplus from trading. The marketing activity becomes the advertisement by the seller in the economy with informational friction. Characteristics and values of the marketing activity are shaped up by the nature of friction. In response to financial friction, a firm can provide financial service to a consumer who would like to borrow against his future income when he purchases an expensive capital good (such as GE Aviation Capital). In response to search friction in a financial market, a financial institution offers brokerage service to match a seller and a buyer of financial assets. The distribution network of a producer is a response to the friction caused by the geographical separation and the limited transportation service. The assessment of brand value depends upon the market structure, especially the nature of frictions in the economy. The brand value typically differs from the present discounted value of the product. The empirical exercise of assessing the brand value requires to identify the nature of friction associated with brand, if multiple forms of friction are present in the market.

We choose an economy with asymmetric information about product quality as a laboratory to investigate our notion of brand for three reasons. First, information asymmetry appears to be the most prominent case in the existing literature. It is not a coincidence that a brand is often used interchangeably with logo or image (de Chernatony and Riley

(1998)), which is crucial for advertisement. Second, we use the model of advertisement (Bagwell (1989)) in order to illuminate the difference between our notion of brand from that of Bagwell (1989). Third, by modeling advertisement as a signaling equilibrium outcome, we can contrast our notion of brand to the existing behavioral and psychological approaches, in the same way as Nelson (1970) departed from the existing views of Bain (1956) and Kaldor (1950) toward advertisement. In a different vein, Kreps (1990) pointed out the reputation as a way to save the transaction cost over time. In a certain sense, our exercise is to show that the brand is a tool to economize the transaction cost under asymmetric information in a one shot game, and to investigate the equilibrium structure of the brand.

We demonstrate that our equilibrium model can answer some puzzles, which cannot be easily explained by the existing approaches. Let us consider a firm with a portfolio of product lines. It is quite common that different products are pooled into the same tier, and the qualities of products in different tiers overlap, as depicted in Figure 1.[1] One may ask why a firm does not want to separate every product out of the same tier by advertising different qualities, as suggested by Milgrom and Roberts (1986). The "overlap" of quality between different tiers appears to cannibalize the consumer base of the same firm, and could hinder the seller from maximizing profit (Aribarg and Arora (2008)). Our equilibrium analysis reveals the precise reason why a profit maximizing firm has incentive to design the products and the tiers with possible overlaps of qualities.[2]

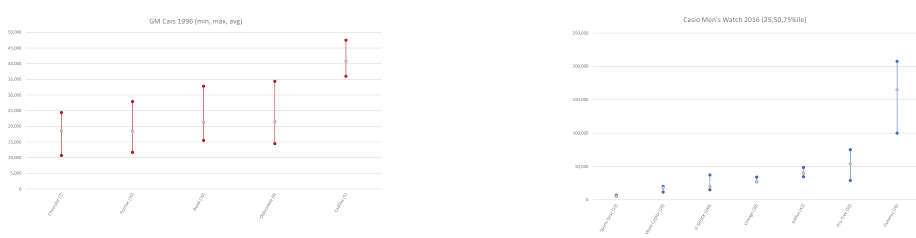

FIGURE 1. Price ranges conditioned on each production line of GM cars (left panel) and men's wrist watch from CASIO (right panel), which we interpret as a proxy for quality. Empty circle in each bar is the mean price. Horizontal axis list tiers according to their orders, and does *not* represent the advertisement expense.

The literature on brand is extensive. In section 2, we review some of the existing studies to motivate the equilibrium analysis of brand. We describe the model formally in section 3, where a monopolistic firm chooses a brand. We call the equilibrium brand an optimal brand. In section 4, we analyze the properties of an optimal brand. In section 5, we conclude the paper with a remark on how our model can be used to determine the social value of a brand, leaving the formal analysis as a future project.

[1]assuming that higher price reflects higher quality of the product.

[2]We derive the conclusion under the assumption that the consumers are homogeneous, and therefore, the firm cannot use horizontal product differentiation.

2. Brief review

Let us examine some existing research on brand, to motivate the need of rigorous equilibrium analysis on brand.[3]

2.1. **Signaling.**

In the existing literature, brand often means a tool for differentiating one's product from the rest, closely related to trademark or advertisement. Following Nelson (1970), a large number of papers (e.g., Milgrom and Roberts (1986), Kihlstrom and Riordan (1984), Bagwell (1989), Bagwell and Riordan (1991), Bagwell (1992) and Bagwell and Ramey (1994)) examined a signaling equilibrium, in which the informed seller credibly signals the quality of the product to a consumer, has been the focus of analysis. A number of studies (e.g., Ackerberg (2003), Erdem, Keane, and Sun (2008), Erdem and Swait (1998), Erdem and Keane (1996), and McDevitt (2014)) explored the empirical implications of advertisement, exploiting the monotonic relationship between the advertisement expense and the product quality in a signaling equilibrium.

By its nature, a signaling equilibrium cannot explain a pervasive structure of brand: multiple products are pooled into the same tier, often assigned with the same trademark, which is then advertised instead of individual products. We need to develop a unified theory to explain the decision by a profit maximizing firm to separate or pool different products. At the same time, we need to completely characterize the observable implications of such an equilibrium, for a class of utility functions often assumed for the empirical analysis.

2.2. **Competitive benchmark.**

By measuring the increment of profit contributed by advertisement, we can estimate the asset value of advertisement (Simon and Sullivan (1993)). The use of the financial data was justified by the complete market hypothesis, under which the financial market price accurately reflects the social value of the brand. However, in a frictionless economy, advertisement carries no social value, since every agent has the same information. Positive social value of advertisement stems from informational friction. To assess the social value of the advertisement, the competitive market must admit informational friction, so that the equilibrium price may not reflect the social value of a product, for example, because of lemon's problem. Using an equilibrium of the complete market or a frictionless economy as a competitive benchmark can lead to a biased estimate of the social value of brand. Data is generated by a market with possibly many forms of friction. We need to identify the specific form of friction associated with the marketing asset of interest, and construct a competitive market *with the same friction*, in order to estimate the social value of brand.

2.3. **Endogeneity.**

Estimation of aggregate demand takes the own market share relative to the no purchase share as the dependent variable, while treating the product characteristics, including prices, as the independent variable. We regard the residual as the brand effect, after regressing away the observed characteristics (e.g., Berry (1994), Berry, Levinsohn, and Pakes (1995), Mazzeo (2002) and Bajari and Benkard (2005)).

[3]It is by no means an extensive or impartial survey of the existing literature on brand. Usual caveats apply.

Endogeneity of prices with respect to the brand effect is a major concern, since it results in bias of the price coefficient. In response, instrumental variables, such as observed product characteristics of rival firms, cost shifters (such as input prices) are used, which are supposed to be orthogonal to the brand effect. Also, to estimate consistently the coefficient on the observed product characteristics, observed characteristics and the brand effects need to be orthogonal, which is hard to justify if we consider brands to provide information on the observed characteristics to consumers.[4]

Erdem and Swait (1998) and Erdem and Keane (1996) regarded brand as the function of consumer perceived characteristics. Researchers who are interested in the effect of advertisements on sales also face the similar endogeneity issues due to the possible correlation between advertising and the residual brand effect, which would be interpreted as the signaling effect.

Selecting a suitable instrumental variable is a tricky task, if the brand effect is an equilibrium outcome. An estimation strategy based on an equilibrium analysis should help researchers select suitable instrumental variables and construct exogenous variations to identify the true signaling effect of advertisement. We shall derive the equilibrium relationship between the observed price and observed advertisement expenditure, which can be tested in the data.

3. Formal description

Our model is inspired by the hedonic model by Rosen (1974). The model defines a product k as a vector

$$\pi_k = (\pi_{k1}, \ldots \pi_{kL})$$

where π_{kj} is the amount of the attribute j in product k. Let us consider a house as an example of product k with two attributes, in which the first attribute of a house is the bedroom and the second attribute is the bathroom. Then, π_{k1} and π_{k2} represent the number of bedrooms and bathrooms, respectively.

We elaborate the basic framework of Rosen (1974) in the two important ways. First, we introduce a rank order among attributes. We order products k such that the products with higher k are of higher quality. We assume that the higher quality products have higher quality attributes. In the example of a house, luxury houses have upgraded bedrooms (with better furniture, higher ceilings etc), and nicer bathrooms. Second, we admit asymmetric information between the seller and a consumer, by assuming that consumers do not have perfect knowledge about the amount of attributes of the product. Thus, the monopolistic firm has to use the advertisement to signal the amount of attributes to the consumers.

We let $\mathcal{K}$ to be the set of all potential products, i.e.,

$$\mathcal{K} = \{1, \ldots, K\}.$$

Each product has at most L attributes. Let

$$\mathcal{L} = \{1, \ldots, L\}$$

[4]Endogeneity also arises in Sullivan (1990), Aaker and Keller (1990) and Sullivan (1998).

be the set of feasible attributes, which is the set of L positive integers. An example of $\mathcal{L}$ would be the list of options. As ℓ increases, the list of options becomes longer. We interpret a higher l as a more desirable attribute.

We then define $\pi_{k\ell}$ as the amount of l attribute in product k. As discussed above, an example of $\pi_{k\ell}$ would be the number of bathrooms if k is a house and ℓ is the bathroom.

We then let $h_{k\ell}$ be the utility a consumer obtains from one unit of the attribute ℓ in product k. Then, we normalize the utility of a consumer so that the marginal utility of the first attribute is $h_{k1} = 0$. For convenience, we normalize the vector π such that $\sum_\ell \pi_{k\ell} = 1$. Then, the set of all feasible products is the unit simplex in $\mathbb{R}^{KL}$, which convex and compact.

To simplify the analysis, while highlighting the informational role of brand, we assume that the production cost of different products is normalized to 0, or has been generated already. This assumption illuminates the precise role of marketing, as a productive activity which incurs after a good is produced but before it is delivered to a consumer. This simplifying assumption in fact strengthens our conclusion by making it easier for an informed seller to separate through the high level or high attribute. Still, we can show that an informed seller chooses to pool different products in an equilibrium.

Let us consider a monopolistic seller, facing a unit mass of infinitesimal consumers. We assume that producer knows the amount of each attribute π_k for each of her product $k = 1, \ldots K$, but the consumers do not. But the consumers know that with probability $\mathcal{P}_\pi(k)$, π_k is realized out of π. We assume that $\mathcal{P}_\pi(k)$ is independent of π, mainly for simplicity. It makes sense to write $\mathcal{P}(k)$ in place of $\mathcal{P}_\pi(k)$.[5] After the firm made a decision on $\{\pi_1, \ldots, \pi_K\}$, it chooses the level of the marketing expense.

Let $G(\pi)$ be the signaling game induced by the profile of products where the type of the sender is π_k with prior $\mathcal{P}(k)$. Let $\mathcal{M} = [0, \infty)$ be the space of messages (or the marketing expenses). Conditioned on π, the sender chooses $m \in \mathcal{M}$ according to

$$\sigma_\pi : \{1, \ldots, K\} \to \mathcal{M}.$$

Conditioned on m, a consumer pays a to purchase one unit of the good. The consumer observes m, but not π_k, before his decision to buy a product. We assume that consumers compete each other in Bertrand fashion to purchase the good, as in the labor market signaling model (Cho and Kreps (1987)).

We choose the signaling space to be a single dimension, while the characteristics of a product is determined by two components, attribute and level. Our modeling feature captures the difficulty that a seller faces in communicating with a consumer. We envision consumers who are intelligent, but can spend only a limited amount of time to pay attention to the advertisement. Because of limited scope of consumers, the message from the seller should be simpler than the actual list of specifications of the product.[6]

While we interpret $m \in \mathcal{M}$ as the amount of marketing expense (or advertisement), we opt for liberal interpretation of m. Instead of a real number, one can view m as an object,

[5] As long as $\mathcal{P}_\pi(k)$ is a continuous function of π, the main result continues to hold.

[6] Our modeling feature captures the language competence of consumers as in Blume and Board (2013), and advertisement as a communication tool. In order to deliver a complex message to a consumer with a limited language competence, the advertisement cannot be precise. The seller finds it optimal choose coarse language to communicate. Our result makes this intuition precise.

which can be ranked in a well defined manner. For example, $\mathcal{M}$ can be a collection of objects ranked according to the perceived cost or a logo to which the firm is known to spend a large sum of money. By choosing intentionally costly object for advertisement, the sender can send a certain signal. Under this liberal interpretation of $\mathcal{M}$, one can include logo in $\mathcal{M}$ as a tool for an advertisement. We call $m \in \mathcal{M}$ a logo for the rest of this paper. We use logo, trademark and advertisement interchangeably. A logo can contain no information about intrinsic properties of the product (Nelson (1970)).

The seller's utility function of a single unit of attribute ℓ with quality level k is

$$u(k, l, m, a) = -f_{kl}m + a$$

and the buyer's utility function of the same is

$$v(k, l, m, a) = h_{kl} - a,$$

where

$$f_{kl} \geq 0 \text{ is strictly decreasing in } k\text{, but strictly increasing in } l \tag{3.1}$$

and

$$h_{kl} \geq 0 \text{ is strictly increasing in } k \text{ and } l \tag{3.2}$$

(3.1) says that the advertisement is costly, whose maginal cost with respect to rank k and attribute l is increasing. We assume linear utility function only to simplify the analysis. The multiplicative separability of the signaling cost of the seller is assumed to facilitate the analysis, while maintaining generality, covering a broad class of models of signaling games.

Given $\pi_k = (\pi_{k1}, \ldots, \pi_{kL})$, we write

$$\pi_k f_k = \sum_l \pi_{kl} f_{kl} \quad \text{and} \quad \pi_k h_k = \sum_l \pi_{kl} h_{kl}.$$

By (3.2), we assume that the advertisement does not affect the intrinsic utility of the buyer, so that the advertisement is socially wasteful, following Milgrom and Roberts (1986).

This is a simplifying assumption that strengthens the conclusion. Even if advertisement per se is socially wasteful in the sense that a consumer obtains on utils from advertisement, advertisement asset may carry a positive social value, under the presence of asymmetric information, in the sense that it will increase the gains from trading.

(3.1) says that the higher the quality level of a product is, the lower the marginal cost of advertisement. On the other hand, it is more costly to signal higher attributes, which formalizes the idea that the more desirable attribute is often more complex, which makes it more difficult to communicate with the consumer. However, the marginal cost of signaling a higher attribute is decreasing as the product rank goes up. That is, it is easier for a luxury line of the product to signal the desirable property than for an entry line of the product, which is known as the single crossing property. (3.2) says that the higher k product has higher quality attributes, and the unit quantity of an attribute with higher l, i.e., more elaborate attributes are more desireable to consumers.

The difficulty of signaling is measured in terms of the marginal rate of substitution between m and a, for a given product (Spence (1973)). Let $\pi_k = (\pi_{kl})_{l=1}^{L}$ be a product and

$$\mathcal{U}_k(m, a) = -\pi_k f_k m + a$$

be the payoff of the seller from selling product π_k and receive a in return, after spending m amount of advertisement. The marginal rate of substitution

$$\left.\frac{da}{dm}\right|_k = \pi_k f_k \tag{3.3}$$

represents the marginal cost for advertisement in return for the marginal increase of revenue. If the marginal rate of substitution is small, one can say that it is relatively easier to signal than otherwise, in the sense of Spence (1973).

The monotonicity of (3.3) with respect to k is known as the *single crossing property*, which is the key condition for the existence and the selection of a separating equilibrium in a signaling game (Cho and Kreps (1987) and Cho and Sobel (1990)). If one assumes that $L = 1$, then the single crossing property holds in our model, since f_k is decreasing in k, and the seller fully reveals the product quality through advertisement as in Milgrom and Roberts (1986).

Since π_k is an endogenous variable, however, the monotonicity of (3.3) with respect to k cannot be assumed. As a result, it is not obvious whether the quality of the product is an increasing function of signal, as in many signaling models of advertisement.

Let π_k be the k-th product line in $\pi = (\pi_k)_{k=1}^K$. Conditioned on m, a buyer forms posterior conjecture $\mu(\pi_k|m)$. We assume that there is a mass of homogenous consumers, who compete as Bertrand competitors (Milgrom and Roberts (1986)). We assume homogenous consumers to eliminate any room for horizontal product differentiation (Berry, Levinsohn, and Pakes (1995)).

Sequential equilibrium is a pair $(\pi, \sigma; \mu)$ of strategy profile (π, σ) and system of beliefs μ, satisfying consistency and sequential rationality (Kreps and Wilson (1982)). In any sequential equilibrium, sequential rationality requires that a consumer pay his expected utility from the consumption, because of Bertrand competition among consumers. Suppose that a consumer has belief about k and π_k conditioned on m as $\mu(\pi_k|m)$, then the payment from the consumer is the expected utility of the product conditioned on m: $a = \sum_{k=1}^K \pi_k h_k \mu(\pi_k|m)$.

In order to ensure that a sequential equilibrium is sustained by a system of reasonable beliefs, we impose a restriction on beliefs off the equilibrium path inspired by criterion $D1$ in each signaling game induced by π (Cho and Kreps (1987) and Cho and Sobel (1990)). We focus on a set of sequential equilibria, in which the possibility of signaling the true type by making a small deviation from the equilibrium logo is exhausted. Fix a particular sequential equilibrium in $G(\pi)$, and m which is not used with a positive probability in the equilibrium. Define

$$\mathcal{D}(m,k) = \{a \mid \mathcal{U}_k(m,a) > \mathcal{U}_k^*\}$$

where $\mathcal{U}_k^*$ is the equilibrium payoff from π_k. $\mathcal{D}(m,k)$ is the set of responses by the consumers, which generates strictly higher profit for π_k than the equilibrium.

Definition 3.1. *A sequential equilibrium satisfies criterion $D1$ if for any m not used in the equilibrium with a positive probability, belief $\mu(\pi_k|m) = 0$ whenever $\exists k'$ such that $\mathcal{D}(m,k) \subset \mathcal{D}(m,k')$. If a sequential equilibrium (σ_π, μ) satisfies criterion $D1$, then we call it a $D1$ equilibrium.*

We define brand as a pair of a product line and the profile of logos associated with the product line.

Definition 3.2. *A brand is (π, m) where $\pi = (\pi_1, \cdots, \pi_K)$ is the profile of products and $m = (m_1, \ldots, m_K)$ is the profile of logos where m_k is assigned to π_k.*

In order to be a meaningful brand, the message should be supported by a $D1$ equilibrium in $G(\pi)$. Otherwise, one product line can make a small deviation from the existing logo to signal its own type, generating higher profit.

Definition 3.3. *Given π and the signaling game $G(\pi)$ induced by $\pi = (\pi_k)_{k=1}^K$, we say that brand (π, m) is valid if $\forall k$, $m_k = \sigma_\pi(k)$ where σ_π is a $D1$ equilibrium in $G(\pi)$.*

Define the ex ante expected profit of a firm from a valid brand induced by π as $\mathcal{U}_s(\pi)$. The firm chooses the profile of products to maximize the profit.

Definition 3.4. *An optimal brand is (π^*, m^*) where m^* is a valid brand in $G(\pi^*)$, and $\mathcal{U}_s(\pi^*) \geq \mathcal{U}_s(\pi)$ $\forall \pi$ and $\mathcal{U}_s(\pi)$ is the equilibrium payoff of a valid brand in $G(\pi)$. From now on, by an equilibrium, we mean an optimal brand.*

Let $D(\pi)$ be the set of $D1$ equilibria in $G(\pi)$. The set of equilibrium payoffs of $D(\pi)$ is known to be upper hemi-continuous with respect to π, from which the existence of a sequential equilibrium of the whole game follows (Cho and Sobel (1990)).

We view the choice over the profile of products is a long term decision by the firm. Given a profile of products, a firm has to choose the level of marketing to signal the quality of the product. In the long run, however, the firm has to configure the product line in order to achieve the maximum profit, given that it will choose the marketing strategic optimally for the profile of products.

Definition 3.5. *Given valid brand (π, m), if $\sigma_\pi(k)$ is one to one so that the seller's product type is fully revealed through different m_k, then we say that the brand is* trivial. *Otherwise, we say that the brand is* non-trivial, *in the sense that different products are under the same brand: $\sigma_\pi^{-1}(k)$ contains more than a single product for some k. If $m = 0$, then we say call (π, m) a generic brand.*

In a frictionless economy, a consumer knows all the relevant attributes of the product. The marketing activity carries no social value. In an optimal brand, $m = 0$. In an economy with asymmetric information, the profit maximizing seller may advertise in an optimal brand. If the attribute of a product is one dimensional (Milgrom and Roberts (1986)), the single crossing property implies $D1$ equilibrium is a separating equilibrium (Cho and Sobel (1990)) and the optimal brand must be trivial.

Let $(m_1, \ldots, m_J)$ be the profile of messages used with a positive probability in an equilibrium: $\forall j$, $\exists k$ such that $\sigma_\pi(k) = m_j$ for $k \in \{1, \ldots, K\}$ and $j \in \{1, \ldots, J\}$ for $J \leq K$. Without loss of generality, assume that $m_1 < \cdots < m_J$.

Definition 3.6. *Fix $G(\pi)$. π_k and $\pi_{k'}$ are in the same tier if $\sigma_\pi(k) = \sigma_\pi(k')$. m is a higher tier than m' if $E[\pi_k h_k \mid m] > E[\pi_k h_k \mid m']$ where m and m' are used with positive probabilities in an equilibrium. We say that a product is in a higher tier than another product, if the expected utility conditioned on the signal is higher.*

Note that the rank of tier is determined by the expected quality of the product in the tier, rather than the amount of advertisement expenditure. If the product has a single dimensional attribute, the single crossing property implies that the expected utility conditioned on the signal is increasing with respect to the signal (Milgrom and Roberts (1986)). In our case, $\forall\pi$, $G(\pi)$ is a monotonic signaling game (Cho and Sobel (1990)), but single crossing property is *not* guaranteed. Consequently, in *valid* brand (π, m), the tier may not increase with respect to the marketing expense m. We shall show that in an *optimal* brand, the products are grouped into tiers, which are ranked according to the amount of marketing expense and the marketing expense is strictly increasing with respect to the tier.

4. Properties of optimal brand

4.1. **Preliminaries.** We shall focus on an optimal brand which entails a finite number of logos, for technical reasons. Let $m = (m_1, \ldots, m_J)$ be the profile of equilibrium messages: $\forall j$, $\exists k$ such that $\sigma_\pi(k) = m_j$ for $k \in \{1, \ldots, K\}$ and $j \in \{1, \ldots, J\}$ for $J \leq K$. Without loss of generality, assume that

$$m_1 < \cdots < m_J.$$

Fix $m \in \{m_1, \ldots, m_J\}$. Define m_- and m_+ as the adjacent logo immediately below and immediately above m:

$$m_- = \max\{m' < m\} \quad \text{and} \quad m_+ = \min\{m' > m\}$$

If $m = m_1$, let $m_- = m$. If $m = m_J$, then let $m_+ = m$. For each $k \in \sigma_\pi^{-1}(m)$, we know the marginal rate of substitution between m and the expected price

$$\left.\frac{da}{dm}\right|_k = \pi_k f_k.$$

Define $\tilde{k}$ as the type which can signal the true type at the minimal cost among those under the same logo m:

$$\left.\frac{da}{dm}\right|_{\tilde{k}} = \pi_{\tilde{k}} f_{\tilde{k}} \leq \pi_k f_k \qquad \forall k \in \sigma_\pi^{-1}(m).$$

Similarly, define $\hat{k}$, as the opposite of $\tilde{k}$, that is the type which bears the largest cost to signal among those under m:

$$\left.\frac{da}{dm}\right|_{\hat{k}} = \pi_{\hat{k}} f_{\hat{k}} \geq \pi_k f_k \qquad \forall k \in \sigma_\pi^{-1}(m).$$

Finally, define $\overline{k}$ as the highest production line among the products under m:

$$\overline{k} = \max \sigma_\pi^{-1}(m).$$

In general, $\overline{k} \neq \tilde{k}$ and $\tilde{k} \neq \hat{k}$.

Let $\tilde{k}_-$, $\hat{k}_-$ and $\overline{k}_-$ be the corresponding elements defined for m_- instead of m. Let p be the unit price of sales of the good under m. Similarly, let p_- be the unit price of sales of the good under m_-.

Note that the expected profit from selling one unit of product π_k under m is

$$\mathsf{E}\left[\pi_k h_k \mid \sigma_\pi(k) = m\right] - m\pi_k f_k = \frac{\sum_{\sigma_\pi(k)=m} \pi_k h_k \mathcal{P}(k)}{\sum_{\sigma_\pi(k)=m} \mathcal{P}(k)} - m\pi_k f_k.$$

4.2. **Signaling cost.** We measure the difficulty of signaling the quality of the product in terms of the marginal rate of substitution between the marketing cost m and the expected return. We show that in an optimal brand, the marginal rate of substitution under the same logo must be the same for all products under the same logo.

In a valid brand, it is possible $\tilde{k} \neq \overline{k}$. But, in an optimal brand, $\tilde{k} = \overline{k}$ in any message used in an equilibrium with a positive probability.

Lemma 4.1. *In an optimal brand* (π, m), $\forall m$ *which is used with a positive probability,* $\tilde{k} = \overline{k}$.

Proof. See Appendix A. □

We can show that every type which sends the same message must have the same marginal rate of substitution between marketing expense m and expected payment a by the consumer.

Proposition 4.2. *Suppose that* π *is the products and* $m_1 < \cdots < m_J$ *are the messages which are sent with a positive probability in the optimal brand. Then, if* $\sigma_\pi(k) = \sigma_\pi(k')$, *then*

$$\left.\frac{da}{dm}\right|_k = \left.\frac{da}{dm}\right|_{k'}.$$

Proof. See Appendix B. □

Proposition 4.2 captures the key feature of the long term decision. In order to maximize profit, the firm designs the individual products so that each product under the same brand m has the same marginal cost of advertisement.

This result does not say, for example, whether $\sigma_\pi^{-1}(m)$ has an *interval* structure: $\exists k' \leq k''$ so that

$$\left\{k \mid \pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}\right\} = \{k' \leq k \leq k''\}.$$

In order to have the interval structure, we need additional conditions.

4.3. **Recursive structure.** By Proposition 4.2, it makes sense to write the marginal rate of substitution as a function of m_i:

$$\gamma_i = \left.\frac{da}{dm}\right|_k \qquad \forall k \in \sigma_\pi^{-1}(m_i).$$

In an optimal brand, a weak form of the single crossing property holds.

Lemma 4.3. *Suppose that* (π, m) *is an optimal brand.* $\gamma_i \geq \gamma_{i+1}$ *if* $m_i < m_{i+1}$.

Proof. See Appendix C. □

Lemma 4.3 is weaker than the single crossing property in the sense that γ_i may not *strictly* increasing with respect to m_i. However, this property is sufficient to ensure that in an equilibrium, we only have to check the incentive of m_i to imitate m_{i+1}. Thus, Lemma 4.3 implies the recursive optimization characterization of the optimal brand.

However, the same result is not sufficient to prove that the expected utility conditioned on m_i is increasing. To this end, we need more results. Recall that we rank the equilibrium messages

$$m_1 < \cdots < m_J.$$

Lemma 4.4. *Suppose that (π, m) is an optimal brand. $\forall i < J$, $\exists j > i$ so that*

$$-\gamma_i m_i + a_i = -\gamma_i m_j + a_j. \tag{4.4}$$

Proof. See Appendix D. □

Lemma 4.4 says that if $\sigma_\pi(k) = m_i$, and $i < J$, then at least one message m_j higher than m_i must satisfy the incentive constraint with equality. To satisfy the incentive constraint of π_k where $\sigma_\pi(k) = m_i$,

$$-\gamma_i m_i + a_i \leq -\gamma_i m_j + a_j$$

must hold for all $j > i$. If the incentive constraint holds with a strict inequality for every $j > i$, then the firm can save the marketing expense by reducing m_j slightly while satisfying the incentive constraint $\forall j > i$, which contradicts the hypothesis that (π, m) is an optimal brand.

Lemma 4.5 strengthens Lemma 4.4, saying that if the incentive constraint of π_k with $\sigma_\pi(k) = m_i$ is binding for m_j as in (4.4), then m_j must be the adjacent message above m_i: $m_j = m_{i+1}$.

Lemma 4.5. *Suppose that (π, m) is an optimal brand. If $\exists j > i$ and*

$$-\gamma_i m_i + a_i = -\gamma_i m_j + a_j,$$

then

$$-\gamma_i m_i + a_i = -\gamma_i m_{i+1} + a_{i+1},$$

Proof. See Appendix E. □

We show that in an optimal brand, the ranking of the advertisement reveals the rank of tiers that is the average quality of the products conditioned on the tier.

Proposition 4.6. *Let (π, m) be an optimal brand where $m = (m_1, \ldots, m_K)$. Given $\pi = (\pi_k)$, m is constructed recursively: m_1 solves*

$$\max_{m_1} \mathsf{E}\left[\pi_k h_k - m_1 \pi_k f_k \mid \sigma_\pi(k) = m_1\right].$$

$\forall i \geq 2$, m_i solves

$$\max_{m_i} \mathsf{E}\left[\pi_k h_k - m_i \pi_k f_k \mid \sigma_\pi(k) = m_i\right]. \tag{4.5}$$

subject to

$$\mathsf{E}[\pi_{k'} h_{k'} \mid \sigma_\pi(k') = m_{i-1}] - m_{i-1}\pi_{k'} f_{k'} \geq \mathsf{E}[\pi_k h_k \mid \sigma_\pi(k) = m_i] - m_i \pi_{k'} f_{k'}$$

$\forall k' \in \sigma_\pi^{-1}(m_{i-1})$. Moreover, $\mathsf{E}\left[\pi_k h_k \mid \sigma_\pi(k) = m\right]$ is strictly increasing with respect to m in an optimal brand.

Proof. Combining the preliminary results, we can characterize the optimal brand recursively. We also know that if $m_i < m_{i+1}$, then

$$-\gamma_i m_i + a_i = -\gamma_i m_{i+1} + a_{i+1}.$$

Thus,

$$a_{i+1} > a_i.$$

Since $a_i = \mathsf{E}[\pi_k h_k | \sigma_\pi(k) = m_i]$, we conclude that the expected quality of m_i is a strictly increasing function of m_i. □

Since the level of advertisement (or logo) is ranked according to the expected quality, we can regard each m_j as a tier of products.

4.4. **Example.** Proposition 4.6 says that the seller *can* differentiate the products through tiers, instead of fully separating individual products (Milgrom and Roberts (1986)). We show through an example that the key implications of Proposition 4.6 are not vacuous. We construct an example in which multiple products must be pooled in the same tier, and the quality of products in different tiers "overlap" in an optimal brand. This example shows that the brand structure depicted in Figure 1 can indeed be optimal, contrary to the claims by some studies (e.g., Aribarg and Arora (2008)).

Proposition 4.7. *Consider the case where $K = 3$ and $L = 2$. That is, we assume three levels of luxuries and two different attributes. Assume that $\mathcal{P}(1) = \mathcal{P}(2) = \mathcal{P}(3) = \frac{1}{3}$. Then, $\exists(f, h)$ satisfying* (3.1) *and* (3.2) *such that in an optimal brand, product lines $k = 1$ and $k = 3$ pool at $m = 0$ and product line $= 2$ separates by choosing $m > 0$.*

Proof. Choose $B \gg A > 0$, and then, choose a small $\epsilon > 0$ to satisfy (3.1) as follows:

$$f_{12} = \frac{B - \beta A}{1 - \beta} > f_{22} = B > f_{32} = A + \epsilon \frac{B - A}{1 - \beta} > f_{11} = A > f_{21} = A - \epsilon > f_{31} = A - 2\epsilon. \quad (4.6)$$

Choose a large $H > 0$ and define

$$\begin{array}{ccccc} h_{11} = 1 + H & < & h_{21} = 1.1 + H & < & h_{31} = 3 + H \\ \wedge & & \wedge & & \wedge \\ h_{12} = 1 + \epsilon^2 + H & < & h_{22} = 2 + \epsilon + H & < & h_{32} = 3 + \epsilon^2 + H \end{array} \quad (4.7)$$

so that (3.2) holds. We need $H > 0$ to ensure that the profit from each product line remains positive.

We make three observations. Define $O(x)$ as "big Oh" function: $\lim_{x \to 0} O(x) = 0$.

- $\pi_1 f_1 = \pi_3 f_3$ requires that $\pi_{11} = 1 - \epsilon$ be close to 1, which in turn implies that $\pi_{31} = O(\epsilon)$, and
$$\pi_{21} = \frac{B - A}{B - A + \epsilon} + O(\epsilon) \simeq 1. \quad (4.8)$$
If the seller wants to pool all three product lines into one tier, product line 2 must have the same marginal rate of substitution as product lines 1 and 3. This constraint requires to assign almost all weight to lower attribute $l = 1$.[7]

[7]This is the feedback from the marketing to the product design, which occurs only in the long run.

- As the attributes increases from $l = 1$ to $l = 2$, $h_{12} - h_{11}$ and $h_{32} - h_{31}$ are $O(\epsilon^2)$, while $h_{22} - h_{21} = 0.9 - \epsilon$ which is strictly bounded away from 0.
- In (4.6), $f_{k2} - f_{k1} > O(\epsilon^2)$ $\forall k$, implying that as the attribute increases, the cost increases faster than the marginal utility of the product, except for product 2.

The remaining step is to compare the expected payoffs of the seller among 3 different types of candidate equilibria: (1) complete pooling at $m = 0$, (2) full separation among three production lines, (3) semi-pooling where (3a) 1 and 2 are pooled and 3 is separated, (3b) 2 and 3 are pooled and 1 is separated or (3c) 1 and 3 are pooled, and 2 is separated. We show that the optimal brand must be a semi-pooling equilibrium where 1 and 3 are pooled, and 2 is separated as in (3c).

In a semi-pooling equilibrium, we have two tiers. We observe that if the tiers are formed consistently with the levels, as in (3a) and (3b), the difference of the expected utility conditioned on the tier is significantly larger than in (3c). By choosing $\epsilon > 0$ sufficiently small, we can make the gain from saving signaling cost outweighs the gain from signaling higher expected utility of the higher tier. The same logic applies to the fully separating equilibrium, which generates the largest signaling cost among all equilibria. We conclude that (3c) generates the largest expected profit of the seller among equilibria that entail some sort of separation (as in (2) and (3)).

The pooling equilibrium generates no signaling cost. However, in order to let $k = 2$ be pooled with $k = 1$ and 3, the seller should choose $\pi_{21} \simeq 1$. That is, in order to make the brand valid, the second level of the product must be a lower quality. Instead, the seller can separate $k = 2$ from the rest of the levels ($k = 1, 3$) by choosing $\pi_{21} = 0$ (that is, making $k = 2$ into a higher quality product). The cost of separation is small, because the average expected utility from lower tier which consists of $k = 1$ and $k = 3$ and the expected utility from the higher tier, consisting solely of $k = 2$ with the higher attribute ($\pi_{22} = 1$). Thus, the gain from separating away from the complete pooling outweighs the cost of signaling, which makes the semi-pooling equilibrium of (3c) the optimal brand. □

4.5. **Structure of an optimal brand.** The example shows that the firm may find it optimal to have "overlap" of quality between two different tiers of products, as shown in Figure 1. But, the same figures show that the seller may want to avoid the overlap, especially at the high end of the product lines.

Recall that a consumer receives higher utility from two sources: higher production line and higher attribute. We show that if the marginal utility of the production line is significantly larger than the marginal utility of the attribute, then the equilibrium brand must have an interval structure. By the same token, if the marginal utility of production line is smaller than the marginal utility of the attributes, then it can be optimal that the equilibrium brand does not have an interval structure. If a consumer cares more about the features of a car rather than the rank of the product, then different automobile brands may have "overlaps," as we have witnessed in the production lines of GM.

In order to investigate the properties across different brands, we impose more structure on f and h. We assume that there exists an increasing function

$$\mu^f : \{1, \ldots, L\} \to \mathbb{R}$$

and a decreasing function

$$\psi : \{1, \ldots, K\} \to \mathbb{R}$$

so that

$$f_{kl} = \psi(k)\mu^f(l).$$

Similarly, there exist increasing function ϕ and μ^h

$$\mu^h : \{1, \ldots, L\} \to \mathbb{R}$$

and

$$\phi : \{1, \ldots, K\} \to \mathbb{R}$$

so that

$$h_{kl} = \phi(k)\mu^h.$$

The (multiplicative) separability of the functional forms of f and h allows us to spell out how the cost and the utility are sensitive to the attributes and the production lines.

Proposition 4.8. *Suppose that $\exists \epsilon^f, \epsilon^h > 0$ such that*

$$\inf_k \phi(k+1) - \phi(k) \geq \epsilon^h, \quad \textit{and} \quad \sup_k \psi(k) - \psi(k+1) \leq \epsilon^f. \tag{4.9}$$

Then, in an optimal brand, $\forall m_1 < m_2$ sent with a positive probability, $\forall k_1 \in \sigma_\pi^{-1}(m_1)$, $\forall k_2 \in \sigma_\pi^{-1}(m_2)$, $k_1 < k_2$.

Proof. See Appendix F. □

The conditions of Proposition 4.8 say that the increment of payoff from choosing high attribute is bounded from below, while the increment of production cost for a higher attribute is bounded from above. In a certain sense, the benefit of signaling higher attributes (generating higher return) dominates the cost of producing goods with higher attributes. The firm finds it optimal to group different products in such a way that it can unambiguously signal the high attributes of products, by eliminating any overlaps of the products across different logos.

4.6. **Different prices.** We have assumed that the good is a credence good so that a consumer does not observe the true quality of a product, in order to simplify the analysis. Our model is static, in the sense that a consumer makes a single decision to buy a good, conditioned on the signal. Since all consumers are homogeneous, a consumer pays up to the expected utility conditioned on the signal, regardless of the actual product. A good example would be the life insurance, which is in fact a complex combination of financial assets, packaged into a single trademark. Depending upon the state such as the financial integrity of the insurance company, the actual utility differs. But, a consumer can observe the quality, only after his death.

We can extend the baseline model in a number of ways, to generate multiple prices for different products under the same trademark.

4.6.1. *Observable attributes.* The seller can add observable attributes as an option. If the seller offers the same set of options for every product in the same tier, offering an option does not reveal the quality of the underlying product. Since a consumer knows exactly the utility from the optional attributes, the price a consumer pays is the sum of the expected utility for the underlying products and the options. An example would be a hotel chain. For example, different hotels under the same trademark, say Holiday Inn, differs in quality, depending upon the location. Yet, a consumer is paying the same price for staying at Holiday Inn, but pays different prices for the options such as a large room and a different level of extra amenities.

4.6.2. *Experience goods.* Following Milgrom and Roberts (1986), we can extend our analysis to cover experience goods, as long as the product quality is revealed slowly to a consumer. Suppose that if a consumer purchase a good in a particular tier conditioned on the message (or trademark), the true quality of the product is revealed to the consumer according to Poisson distribution with probability density $\lambda < \infty$. After the true quality is revealed, the consumer continues to purchase the good, paying the marginal utility of the product, as in Milgrom and Roberts (1986). Since the information is revealed slowly, the firm has incentive to use the information about the product quality strategically, providing the microeconomic foundation for the brand. The baseline model corresponds to the case of $\lambda = 0$. As λ increases, the informational advantage of the seller over a consumer decreases. By the same token, the incentive for imitating the other type of product decreases. Consequently, the size of the marketing cost goes down, as λ increases, although the basic structure of the optimal brand is preserved. If $\lambda = \infty$, the economy no longer has informational friction, and an optimal brand is "no brand" where the seller choose the marketing cost to be 0, regardless of the quality of the product.

5. Concluding remarks

Extensive literature on estimating the brand value (e.g., Simon and Sullivan (1993), Mahajan, Rao, and Srivastava (1994), Swait, Erdem, Louviere, and Dubelaar (1993) and Aaker and Jacobson (1994)) focuses on the market value of brand. On the other hand, a rigorous investigation of the *social* value of a brand appears to be few, which is essential for assessing efficiency of the allocation of the capital, called brand. Unless we have a well defined social value, it is hard to say whether brand is overvalued or undervalued in the market. Our idea to estimate the social value of a brand is closely related to, but differs in important ways from how Simon and Sullivan (1993) and Goldfarb, Lu, and Moorthy (2009) estimated the market value of brand.

5.1. **Challenges.** The market price of brand is determined as an equilibrium outcome of a *market for brand*, which is typically exposed to search and other forms of friction, as the marketing capital is specifically designed for the product. The equilibrium allocation is inefficient, and the equilibrium price of the brand differs from its social value. The literature on decentralized dynamic trading model with information and search friction has matured considerably last decade (e.g., Rubinstein and Wolinsky (1985), Mortensen and Pissarides (1994) and Cho and Matsui (2016)) which offers a menu of useful modeling tools to investigate the price formation process of brand. We shall study the dynamic market for brand after we complete the analysis of the baseline monopolistic model.

Before investigating the decentralized market for brand, however, it is essential to understand the social value of brand, because whether or not the market price is "too high" or "too low" should be judged against the social value of a brand. It would be useful to examine how we can estimate the social value of brand from the market outcome, under the presence of frictions. Existing empirical studies on the price of brand appears to overlook the close link between friction and brand. As a result, complete market is often used as a competitive benchmark (Simon and Sullivan (1993)). The need for identification of the friction associated with brand has drawn little attention.

5.2. **Baseline example.** In order to illustrate the challenges, let us consider a monopolistic market in which the monopolistic firm produces water but is subject to informational friction. Two kinds of water exist, generic (g) and premium (p), which are credence goods. Premium water generates higher marginal utility than generic water, whose marginal utility is normalized to 0. The marginal cost of producing generic water is 0, while the marginal production cost of premium water is $c_p = 7$. If a consumer knows the marginal utility, he pays his marginal utility for one unit of water. Let $w_p = 12$ be the marginal utility of premium water (or the expected payment for premium water). The marginal utility of generic water is normalized to 0.

We assume informational friction. A consumer does not observe the true quality of water, whose prior over the quality of water being premium is 0.5. Since the average quality of water is $0.5(12+0) = 6$ is smaller than the marginal production cost of premium water, lemon's problem prevails, and the premium water producer is not traded.

Suppose that the minimum cost of signaling the premium quality of water is $s_p = 4$. Generic water need not spend money for signaling. The profit from premium water is $\pi_p = [w_p - c_p] - s_p = [12 - 7] - 4 = 1$, while the profit from generic water is 0.

In order to calculate the asset value of a brand from the flow of profit, let us assume that the discount factor is $\delta = 0.9$. The present discount value of the profit of the premium water is $\frac{\pi_p}{1-\delta} = 11 - 0.9 = 10$. s_p is the flow cost of marketing (which in this case is the advertisement). The present discount value of marketing is therefore $\frac{s_p}{1-\delta} = 40$.

5.2.1. *Competitive benchmark.* In order to invoke the counter factual exercise (Simon and Sullivan (1993)), let us consider the market without a signaling mechanism. Lemon's problem takes over, and the premium water will not be traded. Thus, the contribution of the signaling mechanism is to allow the premium water to be traded at the price of 5, minus the cost of signaling, 4. Thus, the social value of the brand should be the present discounted value of the incremental profit 1, which is 10.

In contrast to Simon and Sullivan (1993) and Goldfarb, Lu, and Moorthy (2009), we assume the existence of the informational friction in the competitive benchmark, moving away from the efficient market hypothesis. In a frictionless market in which a consumer knows the quality of water, the premium water will generate profit 5, and therefore, the present discounted value of owning the premium water technology would be 50. One might conclude that the value of the brand is $10 - 50 = -40$ if one uses the frictionless market as a competitive benchmark, which is clearly an under-estimation of the value of the brand.

5.2.2. *Search friction.* Suppose that a potential entrant develops a new technology to produce the premium water at the price of 6.9, lower than the existing firm's cost of 7. Without advertisement, the new firm cannot sell the premium water at the price of 12, because of lemon's problem. Recall that the value of the brand of the existing firm is 10, which is the present discounted value of the flow of profit 1. Following the same reasoning, we know that the value of the brand for the new firm is 11. There is a (social) gain from trading the brand (or the advertisement asset) between the incumbent and the entrant.

The advertisement asset is valuable only for those who have the technology of producing the premium water. Suppose that it takes T rounds before the existing firm is matched to a new firm with a lower production cost technology of the premium water. To highlight the search friction, let us assume that the existing firm's share of the surplus from trading is e^{-T}. That is, if $T = 0$ (i.e., the seller is matched to a buyer immediately), then the brand is sold at the price of 11. If $T = 10$, then the price of a brand will be $10 + e^{-10} \simeq 10$. Because of search friction, the ex ante expected value of the brand must be less than 11, which is the book value of the advertisement asset. In the economy with two sources of frictions, the book value of a brand can be significantly lower than the social value of brand.[8] Unless one can identify the source of friction (information friction) which provides the reason for existence of a brand from the search friction from other friction (search friction), the social value of a brand can be underestimated.

[8]Intangible capital is one possible reason for the gap (Keller (2013)). But, as the intellectual property rights become better established, a certain form of intangible capital such as commercial software can be traded as easily as tangible capital. Intangible capital such as trade secret often lacks a well defined property right. As a result, it is subject to larger friction in the market. It is not intangibility, but friction, that lowers the ex ante value of a capital.

Appendix A. Proof of Lemma 4.1

By the definition, $\tilde{k} \le \overline{k}$. To prove the lemma by way of contradiction, suppose that

$$\tilde{k} < \overline{k}.$$

Then,

$$\pi_{\tilde{k}} h_{\overline{k}} > \pi_{\tilde{k}} h_{\tilde{k}}$$

and

$$\pi_{\tilde{k}} f_{\overline{k}} < \pi_{\tilde{k}} f_{\tilde{k}}.$$

Therefore, $\forall \lambda \in (0,1)$,

$$\hat{p} = \frac{\sum_{k \ne \overline{k}} \pi_k h_k \mathcal{P}(k) + ((1-\lambda)\pi_{\overline{k}} + \lambda \pi_{\tilde{k}}) h_{\overline{k}}}{\sum_{\sigma_\pi(k)=m} \mathcal{P}(k)} > \frac{\sum_{\sigma_\pi(k)=m} \pi_k h_k \mathcal{P}(k)}{\sum_{\sigma_\pi(k)=m} \mathcal{P}(k)} = p.$$

We cannot yet conclude that the deviation from $\pi_{\overline{k}}$ to $(1-\lambda)\pi_{\overline{k}} + \lambda\pi_{\tilde{k}}$ by product $\overline{k}$ generates profit higher than what the seller would have received in the alleged equilibrium. We need to construct a profile of products $\tilde{\pi} = (\tilde{\pi}_k)$, and signaling game $G(\tilde{p})$ and a *valid* brand $(\tilde{\pi}, \tilde{m})$ so that the seller can generate larger profit.

Notice that $\forall \lambda > 0$, $\hat{p} > p$, and $\lim_{\lambda \to 0} \hat{p} = p$. First, consider the indifference curves passing through $(m, \hat{p})$. Locate the indifference curve among those passing through $(m, \hat{p})$, which has the steepest slope. By the definition, it is the indifference curve of $\hat{k}$. Second, locate the indifference curve among those passing through (m_-, p_-) that has the "flatest" slope. By the definition, it is the indifference curve of $\tilde{k}_-$. Finally, note that in order to satisfy the incentive constraint, $\forall k \in \sigma_\pi^{-1}(m)$,

$$\left.\frac{da}{dm}\right|_{\tilde{k}_-} > \left.\frac{da}{dm}\right|_{\hat{k}} \ge \left.\frac{da}{dm}\right|_{k} \tag{A.10}$$

since $m_- < m$.

Choose $(\tilde{m}, \tilde{p})$ with $\tilde{m} > m$, satisfying the two conditions:

$$\hat{p} - m\pi_{\hat{k}} f_{\hat{k}} = \tilde{p} - \tilde{m}\pi_{\hat{k}} f_{\hat{k}} \tag{A.11}$$

$$p_- - m_- \pi_{\tilde{k}_-} f_{\tilde{k}_-} = \tilde{p} - \tilde{m}\pi_{\tilde{k}_-} f_{\tilde{k}_-}. \tag{A.12}$$

(A.11) is the indifference condition for $\hat{k}$, while (A.12) is the indifference condition for $\tilde{k}_-$. By (A.10), $(\tilde{m}, \tilde{p})$ exists and is unique. By the definition of $\hat{k}$, $\forall k \in \sigma_\pi^{-1}(m)$,

$$\hat{p} - m\pi_k f_k \le \tilde{p} - \tilde{m}\pi_k f_k.$$

Thus, every type in $\sigma_\pi^{-1}(m)$ weakly prefers $(\tilde{m}, \tilde{p})$ to (m, p). Recall that $m_- < m < \tilde{m}$. By (A.12),

$$\tilde{p} - \tilde{m}\pi_{k_-} f_{k_-} \le p_- - m_- \pi_{k_-} f_{k_-}$$

$\forall k_- \in \sigma_\pi^{-1}(m_-)$, which ensures the incentive constraint of k_- type.

We can choose $\lambda > 0$ sufficiently small so that $\tilde{m} < m_+$. Define

$$\tilde{\pi}_k = \begin{cases} \pi_k & \text{if} \quad k \ne \overline{k} \\ \tilde{\pi} & \text{if} \quad k = \overline{k} \end{cases}$$

and

$$\tilde{m}_k = \begin{cases} m_k & \text{if} \quad k \ne \overline{k} \\ \tilde{m} & \text{if} \quad k = \overline{k}. \end{cases}$$

We can assign beliefs off the equilibrium path satisfying criterion $D1$. By the construction, $\tilde{m}$ is valid in the induced signaling game $G(\tilde{p})$. Since the seller generates higher payoff in the valid brand $(\tilde{p}, \tilde{m})$ in $G(\tilde{p})$ than the alleged equilibrium, we have a contradiction to the hypothesis that m is valid. This proves our assertion.

Appendix B. Proof of Proposition 4.2

Since m is valid, criterion $D1$ requires that the belief conditioned on $m' \in (m, m_+)$ should be concentrated at

$$\{\pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}\}.$$

Thus, the payoff of $\overline{k}$ from sending m' should be

$$\mathsf{E}\left[\pi_k h_k \mid \pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}\right] - m' \pi_{\overline{k}} f_{\overline{k}}.$$

To be an equilibrium

$$\mathsf{E}\left[\pi_k h_k \mid \pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}\right] - m' \pi_{\overline{k}} f_{\overline{k}} \leq \mathsf{E}\left[\pi_k h_k \mid \sigma^{-1}(m)\right] - m \pi_{\overline{k}} f_{\overline{k}}$$

Since $m' - m > 0$ can be arbitrarily small, the equilibrium condition implies

$$\mathsf{E}\left[\pi_k h_k \mid \pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}\right] \leq \mathsf{E}\left[\pi_k h_k \mid \sigma_\pi(k) = m\right].$$

Recall that $\overline{k}$ is the highest (i.e., most desirable) product type among those k satisfying $\sigma(k) = m$. Thus, the only possibility that the inequality holds is when

$$\{\pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}\} = \sigma_\pi^{-1}(m)$$

which implies that $\forall k \in \sigma_\pi^{-1}(m)$

$$\pi_k f_k = \pi_{\overline{k}} f_{\overline{k}}.$$

Appendix C. Proof of Lemma 4.3

Suppose that $\exists i$ such that $\gamma_i < \gamma_{i+1}$. Since the incentive constraint of m_i must hold,

$$-\gamma_i m_i + a_i \geq -\gamma_i m_{i+1} + a_{i+1}$$

which is equivalent to

$$\gamma_i (m_{i+1} - m_i) \geq a_{i+1} - a_i.$$

Under the hypothesis of the proof, $\gamma_{i+1} > \gamma_i$,

$$\gamma_{i+1}(m_{i+1} - m_i) \geq a_{i+1} - a_i$$

which then implies

$$-\gamma_{i+1} m_i + a_i \geq -\gamma_{i+1} m_{i+1} + a_{i+1}. \tag{C.13}$$

If so, m_{i+1} has incentive to imitate m_i.

Appendix D. Proof of Lemma 4.4

Suppose $\exists i$, $\forall j > i$,

$$-\gamma_i m_i + a_i > -\gamma_i m_j + a_j.$$

$\exists \epsilon > 0$ so that $\forall j > i$,

$$-\gamma_i m_i + a_i > -\gamma_i (m_j - \epsilon) + a_j.$$

That is, we decrease m_j by a small amount, thus saving the marketing cost. By the construction, the incentive constraint for $j > i$ continues to hold, since we simply shift all messages to the left by an equal amount for $j > i$. Also, the incentive constraint of m_i continues to hold, as we choose $\epsilon > 0$ sufficiently small. Thus, the new profile of logos

$$(m_1, \ldots, m_i, m_{i+1} - \epsilon, \ldots, m_K - \epsilon)$$

is valid. Since the seller saves the marketing cost, the profit increases, which contracts the hypothesis that m is a part of an optimal brand.

Appendix E. Proof of Lemma 4.5

Suppose otherwise: $\exists \ell \; i < \ell < j$ so that

$$-\gamma_i m_i + a_i = -\gamma_i m_j + a_j > -\gamma_i m_\ell + a_\ell. \tag{E.14}$$

Incentive compatibility of ℓ implies that

$$-\gamma_\ell m_\ell + a_\ell \geq -\gamma_\ell m_j + a_j. \tag{E.15}$$

By Lemma 4.3,

$$\gamma_i \leq \gamma_\ell \leq \gamma_j. \tag{E.16}$$

Combining (E.14), (E.15) and (E.16), we have

$$-\gamma_j m_j + a_j > -\gamma_i m_\ell + a_\ell \geq -\gamma_\ell m_\ell + a_\ell > -\gamma_\ell m_j + a_j$$

Since $m_j > m_i \geq 0$, we have

$$\gamma_\ell > \gamma_j$$

which contradicts Lemma 4.3.

Appendix F. Proof of Proposition 4.8

We prove a lemma, which reveals precisely where the conditions on ψ and ϕ are used.

Lemma F.1. *$\exists \epsilon^f, \epsilon^h > 0$ such that if*

$$\inf_k \phi(k+1) - \phi(k) \geq \epsilon^h, \quad \textit{and} \quad \sup_k \psi(k) - \psi(k+1) \geq \epsilon^f, \tag{F.17}$$

then $\forall k_1 > k_2$, satisfying $\pi_{k_1} f_{k_1} > \pi_{k_2} f_{k_2}$, $\exists \alpha$ such that if $(\alpha \pi_{k_1} + (1-\alpha)\pi_{k_2}) f_{k_1} = \pi_{k_2} f_{k_2}$, then $(\alpha \pi_{k_1} + (1-\alpha)\pi_{k_2}) h_{k_1} > \pi_{k_2} h_{k_2}$.

Remark F.2. *The substance of this lemma is the existence of the uniform bounds of ϵ^f and ϵ^h. The condition of the lemma says that the marginal utility of k is sufficient large, and the marginal utility of l is sufficiently small. If $k_1 > k_2$, then the cost of signaling should be lower for k_1 than for k_2, for the same product. If $\pi_{k_1} f_{k_1} > \pi_{k_2} f_{k_2}$, then π_{k_1} should assign significantly larger weight to higher attributes than π_{k_2} does. Thus, if one combines the two products, π_{k_1} and π_{k_2}, so that the cost of signaling becomes the same, then the expected payoff from the combined product should be higher than π_{k_2}, thanks to π_{k_1} which assigns large weights to higher attributes.*

Proof. Suppose otherwise. That is, $\exists m_1 < m_2$ and $k_1 > k_2$ such that

$$\sigma_\pi(k_1) = m_1 < \sigma_\pi(k_2) = m_2.$$

Since $k_1 > k_2$ and $m_1 < m_2$,

$$\pi_{k_1} f_{k_1} > \pi_{k_2} f_{k_2} > \pi_{k_2} f_{k_1}.$$

Thus, $\exists \alpha^f \in (0,1)$ so that

$$(\alpha^f \pi_{k_1} + (1-\alpha^f)\pi_{k_2}) f_{k_1} = \pi_{k_2} f_{k_2}. \tag{F.18}$$

A simple calculation shows

$$\alpha^f = \frac{\pi_{k_2} \mu^f (\psi_{k_2} - \psi_{k_1})}{(\pi_{k_1} - \pi_{k_2}) \mu^f \psi_{k_1}}.$$

Since $k_1 > k_2$,

$$\pi_{k_2} h_{k_2} < \pi_{k_2} h_{k_1}. \tag{F.19}$$

If

$$\pi_{k_1} h_{k_1} \geq \pi_{k_2} h_{k_2},$$

it is obvious that

$$(\alpha^f \pi_{k_1} + (1-\alpha^f)\pi_{k_2}) h_{k_1} > \pi_{k_2} h_{k_2}.$$

Suppose that

$$\pi_{k_1} h_{k_1} < \pi_{k_2} h_{k_2} < \pi_{k_2} h_{k_1}.$$

Define $\alpha^h \in (0,1)$ implicitly as

$$(\alpha^h \pi_{k_1} + (1-\alpha^h)\pi_{k_2})h_{k_1} = \pi_{k_2} h_{k_2}$$

which implies

$$\alpha^h = \frac{\pi_{k_2}\mu^h(\phi_{k_2} - \phi_{k_1})}{(\pi_{k_1} - \pi_{k_2})\mu^h \phi_{k_1}}. \tag{F.20}$$

We need to show $\alpha^f < \alpha^h$ uniformly with respect to (π_k) under a certain condition on ψ and ϕ. Let

$$\Pi^* = \{\pi \mid \pi_{k_1} f_{k_1} \geq \pi_{k_2} f_{k_2}\}.$$

Note that

$$\pi_{k_1} f_{k_1} > \pi_{k_2} f_{k_2}$$

implies

$$(\pi_{k_1} - \pi_{k_2})\mu^f \psi_{k_1} > \pi_{k_2}\mu^f(\psi_{k_2} - \psi_{k_1})$$

and therefore,

$$(\pi_{k_1} - \pi_{k_2})\mu^f > \frac{\pi_{k_2}\mu^f(\psi_{k_2} - \psi_{k_1})}{\psi_{k_1}}.$$

By the hypothesis of the proof,

$$\psi_{k_2} - \psi_{k_1} > 0.$$

Since each component of μ_f is strictly positive,

$$\inf \pi_{k_2}\mu_f > 0.$$

Thus, Π^* is a compact set and

$$0 < \inf \frac{\pi_{k_2}\mu^f}{(\pi_{k_1} - \pi_{k_2})\mu^f} \leq \sup \frac{\pi_{k_2}\mu^f}{(\pi_{k_1} - \pi_{k_2})\mu^f} = \overline{x} < \infty.$$

Since each component of μ^h is positive,

$$\inf \pi_{k_2}\mu^h > 0.$$

Define

$$\underline{z} = \inf \frac{\pi_{k_2}\mu^h}{(\pi_{k_1} - \pi_{k_2})\mu^h} > 0.$$

Choose ψ and ϕ so that

$$\sup_k \overline{x}\left[1 - \frac{\psi_{k+1}}{\psi_k}\right] < \inf_k \underline{z}\left[\frac{\phi_{k+1}}{\phi_k} - 1\right].$$

Choose

$$\epsilon^f = \inf_k \left[\frac{\psi_{k+1}}{\psi_k}\right], \text{ and } \epsilon^h = \inf_k \left[\frac{\psi_{k+1}}{\psi_k}\right]$$

which satisfy the above inequality. □

To prove the proposition by way of contradiction, suppose that $\exists m_1 < m_2$, $\exists k_1 \in \sigma_\pi^{-1}(m_1)$ and $\exists k_2 \in \sigma_\pi^{-1}(m_2)$ so that

$$k_1 > k_2.$$

Let $m^e = (m_1, \ldots, m_K)$ with $(m_1 < \cdots < m_K)$ be the equilibrium logos. We say that m_k and $m_{k'}$ are adjacent if $\nexists m' \in \{m_1, \ldots, m_K\}$ $m_k < m < m_{k'}$.

First, we show that we can always find a pair of adjacent logos with the properties assumed by the hypothesis of the proof.

Lemma F.3. *Without loss of generality, we can assume that m_1 and m_2 are adjacent.*

Proof. Suppose that no m_1 and m_2 are adjacent. Let us choose a pair whose distance is minimal:

$$(m_1, m_2) \in \arg\min\{m_2' - m_1' > 0 \mid \exists k_1' \in \sigma^{-1}(m_1'),\ \exists k_2' \in \sigma^{-1}(m_2), k_1' > k_2'\}.$$

By the hypothesis of the proof, m_1 and m_2 are not adjacent to each other. Thus, $\exists m_3$ so that $m_1 < m_3 < m_2$. Moreover, (m_1, m_3) does not have the property of (m_1, m_2): $\forall j \in \sigma_\pi^{-1}(m_3)$, $\forall k \in \sigma_\pi^{-1}(m_1)$, $k < j$.

Since $k_1 \in \sigma_\pi^{-1}(m_1)$,

$$k_1 < j \qquad \forall j \in \sigma_\pi^{-1}(m_3).$$

Since $k_2 < k_1$, we have

$$k_2 < j \qquad \forall j \in \sigma_\pi^{-1}(m_3)$$

even though $m_3 < m_2$. Note that $m_2 - m_1 > m_2 - m_3 > 0$, which violates the definition of (m_1, m_2). □

For the rest of the proof, we assume that m_1 and m_2 are adjacent. Before analyzing a general case, let us start with a special case to illuminate the key idea of the proof.

STEP 1. $\sigma_\pi^{-1}(m_1) = \{k_1\}$. (That is, k_1 is the only product under m_1).

Under the hypothesis of the proof,

$$\pi_{k_1} f_{k_1} > \pi_{k_2} f_{k_2} > \pi_{k_1} f_{k_1}.$$

We can choose $\alpha \in (0,1)$ so that

$$(\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}) f_{k_1} = \pi_{k_2} f_{k_2}.$$

Choose small $\lambda \in (0,1)$. (We later describe how small $\lambda > 0$ should be.) Take $\lambda > 0$ portion of k_1 product, and modify π_{k_1} to $\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}$. By Lemma F.1,

$$(\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}) h_{k_1} > \pi_{k_2} h_{k_2}.$$

Shift the modified product line to m_2. Under this change, m_1 now has $(1-\lambda)\mathcal{P}(k_1)$ production capacity of π_{k_1} product, but m_2 has additional $\lambda\mathcal{P}(k_1)$ capacity.

Let us calculate the amount of change of the expected profit, *if* m^e remains valid.

Remark F.4. *A reader must be warned that after the modification m^e is not valid. But, the expected utility calculation provides a useful benchmark for later analysis.*

Since m_1 only includes π_{k_1}, the unit price of a product under m_1 remains

$$p_1 = \pi_{k_1} h_{k_1}.$$

The loss of sales from m_1 is therefore,

$$\lambda\mathcal{P}(k_1)\pi_{k_1}(h_{k_1} - m_1 f_{k_1})$$

On the other hand, m_2 now has $\lambda\mathcal{P}(k_1)$ portion of $\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}$ product. As a result, the unit price increases from

$$p_2 = \frac{\sum_{\sigma_\pi(j)=m_2} \pi_j h_j \mathcal{P}(j)}{\sum_{\sigma_\pi(j)=m_2} \mathcal{P}(j)}$$

to

$$\hat{p}_2 = \frac{(\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}) h_{k_1} \lambda\mathcal{P}(k_1) + \sum_{\sigma_\pi(j)=m_2} \pi_j h_j \mathcal{P}(j)}{\lambda\mathcal{P}(k_1) + \sum_{\sigma_\pi(j)=m_2} \mathcal{P}(j)}.$$

Thus, m_2 now makes

$$(\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2})(h_{k_1} - m_2 f_{k_1})\lambda\mathcal{P}(k_1)$$

profit more than before. (We have not said that this amount is positive, yet.)

Thus, the total gain of the expected profit from the modification is

$$\lambda\mathcal{P}(k_1)\left[(\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2})(h_{k_1} - m_2 f_{k_1}) - \pi_{k_1}(h_{k_1} - m_1 f_{k_1})\right]$$

Let us consider the term inside of the bracket.

$$\begin{aligned}
& (\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2})(h_{k_1} - m_2 f_{k_1}) - \pi_{k_1}(h_{k_1} - m_1 f_{k_1}) \\
= \;& (\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}) h_{k_1} - m_2 \pi_{k_2} f_{k_2} - \pi_{k_1}(h_{k_1} - m_1 f_{k_1}) \\
= \;& \left[(\alpha\pi_{k_1} + (1-\alpha)\pi_{k_2}) h_{k_1} - \pi_{k_2} h_{k_2}\right] + \left[\pi_{k_2} h_{k_2} - m_2 \pi_{k_2} f_{k_2} - \pi_{k_1}(h_{k_1} - m_1 f_{k_1})\right].
\end{aligned}$$

The term inside of the first bracket is positive by Lemma F.1. The term inside of the second bracket is positive by the incentive constraint between m_1 and m_2. Thus, *if* the monopolist sells the modified production line under the same logo m^e, he will make more profit than in the candidate equilibrium. We have not yet derived the contradiction, since m^e is not valid. In the next step, we make another modification of the production line and construct a new logo $\tilde{m}$ and $(\pi, \tilde{m})$ is valid, but generate the higher level of profit than in the candidate equilibrium, following the same idea as the proof of Lemma 4.1.

STEP 2. Construction of a valid brand.

First, notice that m^e is not valid, since the incentive constraint is not satisfied after the modification is made.

$$\hat{p}_2 - m_2\pi_{k_1}f_{k_1} > p_2 - m_2\pi_{k_1}f_{k_1} = p_1 - m_1\pi_{k_1}f_{k_1}. \tag{F.21}$$

We construct a new profile of logos: $\tilde{M}^e = \{\tilde{m}^1, \ldots, \tilde{m}^{K'}\}$ where $\tilde{m}_k = m_k$ if $m_k \neq m_2$. Only change is made to m_2, in order to satisfy the incentive constraint. Choose $(\tilde{m}_2, \tilde{p}_2)$ according to

$$\hat{p}_2 - m_2\pi_{k_2}f_{k_2} = \tilde{p}_2 - \tilde{m}_2\pi_{k_2}f_{k_2} \tag{F.22}$$

$$\tilde{p}_2 - \tilde{m}_2\pi_{k_1}f_{k_1} = p_1 - m_1 f_{k_1}\pi_{k_1}. \tag{F.23}$$

(F.22) says that m_2 is indifferent between $(m_2, \hat{p}_2)$ and $(\tilde{m}_2, \tilde{p}_2)$. Similarly, m_1 is indifferent between (m_1, p_1) and $(\tilde{m}_2, \tilde{p}_2)$. The solution exists and unique, because

$$\pi_{k_1}f_{k_1} > \pi_{k_2}f_{k_2}.$$

Note that the amount of change is a continuous and monotonic function of $\lambda > 0$. Let

$$m_{2+} = \inf\{m_k \mid m_k > m_2\}$$

be the adjacent logo above m_2. Choose $\lambda > 0$ sufficiently small so that

$$\tilde{m}_2 < m_{2+}.$$

$\tilde{m}$ is the profile of logos obtained by replacing m_2 by $\tilde{m}_2$. By the construction, $(\pi, \tilde{m})$ is valid, since the incentive constraint is binding. It is straightforward to assign beliefs off the equilibrium path satisfying criterion $D1$. We now show that the total expected profit from $\tilde{m}$ is larger than from m^e, thus deriving the contradiction.

Let $\tilde{\lambda}$ be the proportion of k_1 product to move from m_1 to $\tilde{m}_2$ to achieve the unit price (which is the expected return from the good) $\tilde{p}_2$.

The profit gain from $\tilde{M}^e$ over m^e is

$$(\tilde{p}_2 - \tilde{m}_2\pi_{k_2}f_{k_2})\left(\tilde{\lambda}\mathcal{P}(k_1) + \sum_{\sigma_\pi(j)=m_2}\mathcal{P}(j)\right)$$
$$+(1-\tilde{\lambda})\mathcal{P}(k_1)(p_1 - m_1\pi_{k_1}f_{k_1}) - (p_2 - m_2\pi_{k_2}f_{k_2})\sum_{\sigma_\pi(j)=m_2}\mathcal{P}(j). \tag{F.24}$$

By the construction of $(\tilde{m}_2, \tilde{p}_2)$,

$$\tilde{p}_2 - \tilde{m}_2\pi_{k_2}f_{k_2} > p_2 - m_2\pi_{k_2}f_{k_2}.$$

Since $\pi_{k_2}f_{k_2} < \pi_{k_1}f_{k_1}$, and since the incentive constraint of m_1 is binding at $(\tilde{m}_2, \tilde{p}_2)$,

$$\tilde{p}_2 - \tilde{m}_2\pi_{k_2}f_{k_2} > \tilde{p}_2 - \tilde{m}_2\pi_{k_1}f_{k_1} = p_1 - m_1\pi_{k_1}f_{k_1}.$$

Therefore, (F.24) is positive, contradicting the hypothesis that (π, m^e) is an optimal brand.

STEP 3. General case.

Let us consider a general case where $\sigma_\pi^{-1}(m_1)$ may contain multiple products. Without loss of generality, we can assume that

$$k_1 = \max \sigma_\pi^{-1}(m_1).$$

The key difference from the special case where $\sigma_\pi^{-1}(m_1)$ is singleton is that as we take $\tilde{\lambda}\mathcal{P}(k_1)$ mass from m_1 to m_2, the unit price of m_1 changes from

$$p_1 = \frac{\sum_{\sigma_\pi(j)=m_1} \pi_j h_j \mathcal{P}(j)}{\sum_{\sigma_\pi(j)=m_1} \mathcal{P}(j)}$$

to

$$\tilde{p}_1 = \frac{\sum_{\sigma_\pi(j)=m_1} \pi_j h_j \mathcal{P}(j) - \lambda \mathcal{P}(k_1)\pi_{k_1} f_{k_1}}{\sum_{\sigma_\pi(j)=m_1} \mathcal{P}(j) - \lambda \mathcal{P}(k_1)}.$$

Let m_{1-} be the adjacent logo of m_1 while $m_{1-} < m_1$. Let $\sigma_\pi(k_{1-}) = m_{1-}$. We choose $(\tilde{m}_2, \tilde{p}_2)$ according to

$$\hat{p}_2 - m_2\pi_{k_2} f_{k_2} = \tilde{p}_2 - \tilde{m}_2\pi_{k_2} f_{k_2} \tag{F.25}$$

$$\tilde{p}_2 - \tilde{m}_2\pi_{k_1} f_{k_1} = \tilde{p}_1 - \tilde{m}_1 f_{k_1}\pi_{k_1}. \tag{F.26}$$

where $\tilde{m}_1 < m_1$ is selected to satisfy the incentive constraint of m_{1-} with equality:

$$p_{1-} - m_{1-}\pi_{k_{1-}} f_{k_{1-}} = \tilde{p}_1 - \tilde{m}_{1-}\pi_{k_{1-}} f_{k_{1-}}.$$

One can easily show that $(\tilde{m}_1, \tilde{p}_2)$ and $\tilde{m}_{1-}$ exist uniquely. We can apply exactly the same logic after constructing $\tilde{M}^e$ after replacing m_2 by $\tilde{m}_2$ as before, and replacing m_1 by $\tilde{m}_1$.

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