Uncertainty-driven Cooperation*

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Abstract

We consider a team of agents working on a joint project with unknown true prospects. Team members receive interim feedback that is informative for effort choices and the project's prospects. We show that the presence of uncertainty alleviates inefficiencies arising from free-riding. The team members exaggerate their efforts to influence the interim feedback signal, which in turn affects their partners' beliefs about the uncertain state, consequently affecting their future effort choices. The equilibrium effort level can be approximately efficient when feedback is sufficiently responsive. Our result implies that creating uncertainty in team projects could lead to a Pareto improvement. We also study an asymmetric information model in which some of the agents know the true prospects while the others are uninformed.

Keywords: Team Production, Free-riding, Dynamic Games, Uncertainty, Learning

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1 Introduction

Team production and the use of group incentives are increasingly widespread in the modern workplace.[1] The benefits of team production and its suitability for modern industries have been widely acknowledged. Yet, team production suffers from the well-known free-riding problem. Consequently, determining ways to mitigate free-riding has gained practical and theoretical importance. The literature has generally suggested that cooperation can be sustained by "punishments" based on past behavior, in the form of either lower monetary transfers or future non-cooperation by other team members.[2]

We depart from the classical punishment-based mechanisms by identifying a new factor that mitigates free-riding in teams: *the presence of uncertainty*. Team projects often entail uncertainty about their true prospects: In entrepreneurial projects, no member in a start-up company can precisely predict how successful their firm will be; in academic writing, researchers do not know whether and how their initial hypothesis will be confirmed. Clearly, there is always a standard cost of uncertainty resulting from uninformed action choices. In this paper, we identify a *benefit* of uncertainty that appears in dynamic team production environments. Specifically, we demonstrate that such uncertainty inflates the agents' incentives to exert effort, thus countering the impact of free-riding.

We consider a team of agents working on a joint project with unknown true prospects. The agents share the common output of the project, and thus, the free-riding problem inherently exists. We assume that an agent's effort level is unobservable to others. Over time, the agents receive interim public feedback about their performance. This feedback is noisy but informative about the agents' efforts and the project's prospects: The agents are likely to receive good feedback if they contribute high effort, and the feedback is positively correlated with the prospects of the project.

In this environment, the presence of uncertainty provides each team member with another

[1] Using survey data, Osterman (1994, 2000) estimates that among private, for-profit establishments that have at least 50 employees, approximately 40% have at least half of their employees organized in teams. Similarly, Lawler et al. (2001) reports that 47% of Fortune1000 companies make use of self-managed teams.

[2] In the literature on contracts with many agents, a group contract based on the total output can mitigate moral hazard in teams (Holmström, 1982; Legros and Matthews, 1993); in repeated partnership games, the threat of future non-cooperation following a deviation sustains various equilibrium dynamics (Radner et al., 1986).

type of benefit from exerting effort, in addition to the usual (myopic) individual benefit. This new benefit arises because working harder today improves the interim feedback, rendering team members more optimistic about the project. Optimistic agents exert more effort in the subsequent phases because better prospects provide the agents with higher marginal benefit of effort. The returns from this increased future effort accrue to all team members, including the one who exerted more effort. Essentially, the presence of uncertainty endogenously generates "strategic complementarity" between one agent's current effort and the others' future efforts.[3] This strategic complementarity leads to an equilibrium effort level that is higher than the myopically optimal level.[4]

The above argument directly leads to another important message: *It could be beneficial to create uncertainty in team production environments*. While uncertainty always entails a cost resulting from mismatched action choices, the benefit from mitigating the free-riding problem could outweigh the cost. In this case, adopting a project with uncertain prospects—even without the possibility of high returns—would lead to a Pareto improvement for the team members.

Our results have implications for entrepreneurial behavior in firms and organizations. Risk-taking is considered one of the main elements of entrepreneurial behavior (Miller, 1983). The economic literature has suggested various motivations for risk-taking behavior, such as the desire to receive a higher premium (Heaton and Lucas, 2000) or the desire to smooth out the entrepreneur's value as a function of wealth (Vereshchagina and Hopenhayn, 2009). In this paper, we identify an alternative motivation by demonstrating that risk-taking can benefit organizations by mitigating the free-rider problem.

Our main model features a unique perfect Bayesian equilibrium. The analysis of the unique equilibrium leads to the following additional conclusions.

[3] By strategic complementarity, we refer to the following: Fix all agents' expectations about the effort choice of a given agent. Then, *provided that the effort is unobservable*, an increase in the agent's effort level would lead to an increase in the expected marginal return on the other agents' future efforts. Although this is not precisely the standard definition of strategic complementarity (Bulow et al., 1985), we will use the term with some abuse, as we believe that it effectively captures the essence of the mechanism we identify.

[4] This mechanism can be interpreted as a novel application of "signal-jamming," which has been identified in various contexts including early work in industrial organization (Riordan, 1985; Fudenberg and Tirole, 1986) and the seminal paper of Holmström (1999) in the context of agency theory. We further discuss our contribution relative to these models in the literature review below.

- *Equilibrium can be approximately efficient*: We show that there exists an upper bound that if the feedback is sufficiently responsive to the agents' efforts, then the equilibrium effort level is arbitrarily close to this upper bound for nearly the entire duration of play (Proposition 8). Furthermore, we show that there exists a share structure under which the upper bound coincides with the socially efficient level. This implies that under the right share and feedback structures, the unique equilibrium is approximately efficient and the free-riding problem is almost entirely eliminated.

- *Imperfect monitoring is essential*: Unsurprisingly, if individual effort choices are perfectly observable, then the signal-jamming incentives disappear and the agents choose the myopically optimal effort level (Proposition 3). From the perspective of team design, this result implies that in environments in which monitoring is costly, it may be beneficial to choose a project with uncertain prospects instead of investing in the monitoring structure and establishing formal contracts.

- *Incentives are stronger under more flexible actions*: We show that in a stationary environment, the equilibrium effort level is more exaggerated if actions are more flexible—that is, if agents can more frequently change their effort levels and receive feedback (Proposition 11). This result provides an interesting contrast to the repeated partnership literature, in which the scope of cooperation could be limited when actions are flexible (Abreu et al., 1991; Sannikov and Skrzypacz, 2007).

We also consider several extensions and applications of our model. First, we consider an infinite-horizon version in which the true prospects vary stochastically over time. Second, we consider an asymmetric information model in which some team members are "experts" who are perfectly informed about the prospect. We show that the essential structure of the unique equilibrium of our model extends to these cases. Additionally, using the asymmetric information model, we discuss the interaction between the incentives to signal (for the experts) and to signal jam (for the uninformed agents). Finally, we present applications of our model regarding optimal team design. In particular, we consider the impact of the presence of experts on equilibrium behavior and demonstrate that the expected surplus could be nonmonotonic in the number of experts on a team. We also consider the question of the optimal formation of

		2		
		H	M	L
1	H	$4p$	$3p$	$2p$
	M	$3p$	$2p$	p
	L	$2p$	p	0

		2		
		H	M	L
1	H	$2p_G+2p$	$2p_G+p$	p_G+p
	M	$2p_G+p$	$2p_G$	p_G
	L	p_G+p	p_G	0

Table 1: Success probability when $\theta = \mathit{Bad}$ (left) and *Good* (right)

teams when agents have heterogeneous costs.

1.1 A Two-period Example

Many of our main insights can be explained by a simple two-period, two-type model. Two agents $i = 1,2$ engage in team production over two periods $t = 1,2$. In each period, each agent chooses an effort level $a_i \in \{L,M,H\}$, which is interpreted as contributing zero, one, or two units of effort, respectively. The cost of effort is $c(L) = 0$, $c(M) = c$, and $c(H) = 2c$. We assume that the effort level of each agent is unobservable to the other agent. A stochastic outcome, either *success* or *failure*, is revealed at the end of each period. If success is revealed, each agent receives a payoff of one. In the event of failure, a zero payoff is received. The agents have a common discount factor $0 \leq \delta \leq 1$.

The probability of success in each period depends on both the project type θ, which can be either good (G) or bad (B), and the stage action profile. Table 1 depicts the success probability for each project type. When the project is bad, each unit of effort increases the success probability by p; when the project is good, the first and the second unit of effort raise the probability by p_G and p, respectively. We assume that $p < c < p_G$, and hence, when the type is known, the equilibrium in the stage game differs by project type: For the bad project, L is the dominant strategy of the stage game (since $c > p$); if the project is good, M is the dominant strategy (since $c < p_G$). We further assume that $c < 2p$, and thus, the free-riding problem exists regardless of the project type: H is the socially desirable action for both project types. Note that if the project type is known, in the two-period game, the agents play (M,M) if $\theta = G$ and (L,L) if $\theta = B$ in both periods.

Now, consider incomplete information about the project type: Let μ_t be the common period-t belief that $\theta = G$. At $t = 2$, each agent plays the myopically optimal strategy: An agent exerts effort only if its individual benefit is greater than its cost. As a result, the agents play (M,M) if $\mu_2 > \mu^*$ and (L,L) if $\mu_2 < \mu^*$, where $\mu^* = \frac{c-p}{p_G-p}$.

In the first period, however, the agents' incentives also depend on the effect of the current action on the future belief, as the agents' effort choices affect the probability of success. The next proposition states that this additional effect could boost the agents' effort level in equilibrium, even possibly to a level higher than that when the project is known to be good.

Proposition 1. *Suppose that $p+\delta pc > c$. Then, there exist $\underline{\mu} < \mu^* < \bar{\mu}$ such that for any $\mu_1 \in (\underline{\mu}, \bar{\mu})$, there is a unique sequential equilibrium. In this equilibrium, the agents play (H,H) in $t=1$.*[5]

The intuition is explained by a multiagent version of the signal-jamming effect. Suppose that the prior belief μ_1 equals μ^*. Note that the outcome in the first period changes the agent's belief about the project type, leading to different best responses in period 2. After success in period 1, the agents would become more optimistic about the project type ($\mu_2 > \mu_1 = \mu^*$), and thus, they would play M in period 2; if failure is realized, then the agents would play L, as $\mu_2 < \mu^*$. However, each agent wants the other to exert more effort, regardless of the state. Thus, each agent has an additional incentive to increase his effort level in $t=1$, as a higher effort level leads to a higher success probability. For example, if an agent increases his effort level from M to H, then his marginal benefit consists of the myopic benefit and the benefit from "jamming" the first-period signal:

$$\underbrace{p}_{\text{myopic benefit}} + \underbrace{\delta \times p \times (\mu^*(2p_G - c) + (1-\mu^*)(2p-c))}_{\text{benefit from higher effort in } t=2} = p + \delta pc.$$

As a result, if $p+\delta pc > c$, then choosing H in the first period is each agent's optimal strategy. Note that each agent expects the other's action in equilibrium, and thus, each agent is actually confined to a high effort level because contributing lower effort makes the other agent more pessimistic about the project.

The next corollary states that the benefit from the signal-jamming effect is so strong that creating *pure downward risk* could be welfare-improving: The agents (or an imaginary prin-

[5]The proof of the proposition is in the Online Appendix.

cipal who hires the team of agents) are better off by choosing a project that could be good or bad over a project that is good for certain.

Corollary 1. *Suppose that $p + \delta pc > c$. Then, there exists $\hat{p} > c$ such that $p^G < \hat{p}$ if and only if the agent's expected payoff under the unique sequential equilibrium when $\mu = \mu^*$ is higher than the equilibrium payoff when $\mu = 1$.*

When the horizon is longer than two periods, the signal-jamming effect becomes stronger and the benefit of creating uncertainty becomes greater. Moreover, the agents' incentives are affected by the fact that the agents' beliefs might diverge off the equilibrium path. Not only does such belief divergence play a crucial role in the equilibrium dynamics, but it also complicates the equilibrium analysis. Nevertheless, we specify a tractable model that allows us to analyze such interactions and thus enables us to investigate the equilibrium dynamics in a longer-horizon team production game. Moreover, within this model, we are able to analyze various applications, such as comparative statics with information parameters and extension to the case with asymmetric information.

The remainder of the paper is organized as follows. The remainder of this section discusses related literature. Section 2 formally describes the model. Section 3 characterizes the equilibrium and undertakes comparative statics exercises. Section 4 analyzes an infinite-horizon version of the model with a stochastic state. Section 4.1 extends the analysis to the case with asymmetrically informed agents. Section 5 addresses applications. Section 6 concludes. The proofs for the main results are in the Appendix. The rest of the omitted proofs are in the Online Appendix, which also discusses the potential non-monotonicity of the belief sensitivity and constructs and analyzes the continuous-time version of the main model.

1.2 Literature Review

This paper contributes to the literature on free-riding in groups (Olson, 1965; Alchian and Demsetz, 1972; Holmström, 1982). The literature has identified inefficiencies of team production when the members share the output and has investigated several remedies for this problem. Our paper analyzes dynamic moral hazard in team production in the presence of uncertainty and demonstrates that the presence of uncertainty over a project's prospects could alleviate free-riding.

Our paper is related to the literature on experimentation in teams. The literature focuses on the effect of either a pure informational externality (Bolton and Harris, 1999; Keller et al., 2005; Rosenberg et al., 2007) or combinations of information and payoff externalities (Bonatti and Hörner, 2011; Halac et al., 2016; Guo and Roesler, 2016). In contrast, our model considers a *pure payoff externality*: We specify our model such that the speed of learning is independent of the agents' actions, and consequently, the agents do not have incentives for experimentation. Moreover, in the literature, the presence of uncertainty is necessary for inefficient outcomes (the equilibrium is socially efficient if the state is known), whereas in our paper, the uncertainty mitigates the free-riding problem and therefore improves efficiency.

In this literature, the closest papers to ours are Bolton and Harris (1999) and Bonatti and Hörner (2011). Bolton and Harris (1999) consider a multi-agent experimentation problem in which agents' actions are observable and the agents share the information, but not the payoff, resulting from experimentation. Their symmetric Markov perfect equilibrium demonstrates that the possibility of eliciting future experimentation by others encourages current experimentation. While our unique equilibrium exhibits similar incentives, the underlying channels are distinct. Whereas in Bolton and Harris (1999), the agents are encouraged to demonstrably generate new information (*convincing*), in our model, the agents' incentives are generated by secretly manipulating the feedback (*cheating*). Indeed, for the "encouragement effect" to exist in our case, it is essential that the agents' effort choices are unobservable. Bonatti and Hörner (2011) consider dynamic moral hazard in teams with an uncertain state. In their paper, the game ends when the common project has a "breakthrough," the arrival rate of which depends on the agents' current effort levels and the unknown quality of the project. This instantaneity of potential success implies that one agent's current effort and the others' future efforts are strategic substitutes, leading to inefficiencies in the form of procrastination. In contrast, in our model, uncertainty over the project's prospects generates a form of strategic complementarity between an agent's current effort and the others' future efforts, strengthening the incentives to exert effort and sometimes leading to an (approximately) socially efficient outcome.

Our paper is also related to the literature on dynamic contributions to public goods. Admati and Perry (1991) and Marx and Matthews (2000) show that a public project can be completed by agents who contribute small amounts from time-to-time. Yildirim (2006) and Georgiadis (2014) assume that the payoff is realized only when the project's state reaches a pre-specified threshold. In these papers, the threshold-payoff assumption implies that the effort choices at different points in time are strategic complements, which plays a key role in mitigating the free-riding problem. Importantly, these papers do not feature uncertainty over the project's type. In contrast, our repeated partnership game does not assume the completion threshold, and the complementarity between current and future efforts arises because the agent's effort affects the other's inferences.[6]

As noted above, the signal-jamming mechanism of our paper has been investigated in various contexts. Since Holmström (1999), the literature on career concerns has analyzed the "market-based" incentives of a manager who attempts to affect the market belief about his innate ability. Riordan (1985) (oligopoly) and Fudenberg and Tirole (1986) (entrant-incumbent game) consider cases in which a firm has a signal-jamming incentive to make the competing firm more pessimistic about future profitability. Cisternas (2016b), using the first order approach, expands the career-concerns model to allow more general payoffs for the long-run agent. He demonstrates that the stationary equilibrium exhibits "ratcheting" behavior.[7] We contribute to this literature by identifying the role of such mechanisms in team production. In addition, we fully describe the evolution of such ratcheting in our non-stationary model. Moreover, we identify a novel and important implication of ratcheting: Due to its existence, agents' incentives remain bounded even as the horizon becomes long. This is in contrast to the outcome in Holmström (1999)'s environment, where the optimal efforts of the agents may diverge.

Our paper (especially the infinite-horizon model in Section 4) is related to the literature on repeated games with frequent actions. Starting with Abreu et al. (1991), the literature shows how frequent actions can be detrimental to cooperation (Sannikov and Skrzypacz, 2007; Fudenberg and Levine, 2007, 2009). In contrast, we show that the frequent actions in our

[6] See also Georgiadis (2016) for how deadlines and the frequency of the monitoring affect free-riding incentives.

[7] Cisternas (2016a) generalizes the career concerns model in another dimension allowing for human capital investments.

model increase the level of cooperation.

2 Model

A team of N agents is working on a common project. Time $t = 0, \dots, T$ is discrete and finite. Each period has length $\Delta > 0$, and $\tau = T\Delta$ is the real-time length of the project. At the beginning of the game, nature draws a persistent state of the world θ from a Gaussian distribution $\mathcal{N}(\mu_0, 1/\nu_0)$, which defines the initial common prior about θ.[8] In each period, agent i chooses an effort level $a_{it} \in \mathbb{R}$. Each agent's effort choice is not observable by other agents. We assume that agent i incurs a quadratic cost of effort $\Delta c_i a_{it}^2/2$, where $c_i > 0$. The agents have a common discount factor $\delta = e^{-r\Delta}$, where $r > 0$.

At the end of each period, the agents publicly observe feedback y_t. This can be the outcome of an internal review or feedback from an employer. We assume that the period-t feedback is

$$y_t = \Delta \left[\kappa_\theta \theta + \kappa_a \sum_{i=1}^{N} a_{it} + \varepsilon_t \right],$$

where $\varepsilon_t \sim \mathcal{N}(0, 1/\nu_\varepsilon)$ is a stochastic noise term with precision $\nu_\varepsilon = \Delta\eta_\varepsilon$ and $\kappa_\theta, \kappa_a > 0$ are positive constants. We interpret η_ε as the information disclosure rate. Note that the informativeness of each feedback increases in Δ.[9] The parameters κ_a and κ_θ determine how sensitive the feedback is to agents' actions and to the realization of the true state, respectively. We assume that ε_ts are independent and identically distributed over time.

Total production P is realized at the end of period T and is given by

$$P = e^{r\tau} \sum_{t=0}^{T} e^{-rt\Delta} P_t,$$

where $P_t = \Delta\theta \sum_{i=1}^{N} a_{it}$ is period-t production and $e^{r\tau}$ is a normalization term.[10] Note that P_t is

[8] In Section 4, we extend our result to the case of a stochastic state.

[9] As $\Delta \to 0$, a linear interpolation of the feedback process y_t converges in distribution to $dY_t = \left(\kappa_\theta \theta + \kappa_a \sum_{i=1}^{N} a_{i,t}\right) dt + \frac{1}{\sqrt{\eta_\varepsilon}} dW_t$, where W_t is a standard Brownian motion (Whitt, 1980).

[10] Our assumption that output is realized at the end of the game is not essential for our main mechanism, as demonstrated in our two-period example. This assumption allows the linear feedback to be the only source of belief updating throughout the game and thus significantly simplifies the analysis. Nevertheless, one can find various real-world examples in which the returns to effort are realized at a specific future date, such as the release

linear in each agent's period-t effort and the state θ is the marginal product. Further note that in this specification, output is additively separable in effort across agents and over time.

The agents share the total production according to a rule $(s_1, \cdots, s_N)$, where s_i represents agent i's share of the total output with $\sum_{i=1}^{N} s_i = 1$ and $s_i > 0$ for all i. The agents are risk-neutral expected utility maximizers, with agent i maximizing

$$\begin{aligned} U &= \mathbb{E}\left[s_i e^{r\tau} P - \sum_{t=0}^{T} e^{-rt\Delta} \Delta c_i \frac{a_{it}^2}{2} \right] \\ &= \sum_{t=0}^{T} \Delta e^{-rt\Delta} \mathbb{E}\left[s_i \theta \sum_{j=1}^{N} a_{jt} - c_i \frac{a_{it}^2}{2} \right]. \end{aligned}$$

Remark 1. The agent's payoff in our model is *not* additively separable in the agent's action (a_{it}) and the state (θ). Such complementarity between the action and the state is crucial for generating our main result. Without this complementarity, the agent's marginal benefit of effort—and therefore the optimal effort level—would be independent of the state, and thus, the incentive to manipulate others' beliefs would disappear.

Remark 2. The agents' action and the state enter in an additively separable way into the feedback y_t. As our introductory two-period example shows, such additive separability is not necessary for our results, but it renders our dynamic model very tractable. In particular, as Section 3 clarifies, this assumption implies that the speed of learning is independent of agents' actions, and thus, the agents in our model do not have incentives for experimentation. This makes the underlying mechanism of the model different from those in the literature on experimentation in teams (Bonatti and Hörner, 2011; Keller et al., 2005).

A public history $h^t \in \mathscr{H}$ is a feedback sequence $\{y_k\}_{k=0}^{t-1}$. Agent i's private history $h_i^t \in \mathscr{H}^i$ is the combination of the public history and the sequence of his own past effort choices, that is, $h_i^t = \{(a_{ik}, y_k)\}_{k=0}^{t-1}$.[11] A pure strategy for agent i is a function $a_i : \mathscr{H}_i \to \mathbb{R}$, where $a_{it} = a_i(h_i^t)$ is agent i's effort level in period t. We focus on pure strategy profiles.

The solution concept is perfect Bayesian equilibrium (PBE).[12] A PBE is a strategy profile

of a new product or the issuance of an IPO.

[11] As usual, we define $h^0 = h_i^0 = \emptyset$ for all i.

[12] For the formal definition of PBE, see Fudenberg and Tirole (1991) Definition 8.2.

$a = (a_1, \dots, a_N)$ and a belief system such that the beliefs on and off the equilibrium path are derived using Bayes' rule from the strategies whenever possible, and each player's strategy is optimal given his beliefs and the strategies of others.

Benchmark cases We conclude this section by considering two benchmark cases for future reference. The proofs are straightforward and thus omitted.

1. Static setting ($T = 0$): Agent i's effort in the unique equilibrium of the static setting is $a^*_{i,static} = \mathbb{E}[\theta] = \frac{s_i}{c_i}\mu_0$. Note that the socially efficient level of effort (the one without the free-riding effect) is $\frac{1}{c_i}\mu_0$.

2. Complete information case ($\nu_0 = \infty$): Suppose that the state of the world θ is fully known. Then, the unique equilibrium profile is $a^*_{it} = \frac{s_i}{c_i}\theta$ for any $t = 0, \dots, T$, while the socially efficient level is $\frac{1}{c_i}\theta$.

3 Equilibrium

In this section, we derive the unique PBE of our model. We also discuss the resulting equilibrium dynamics and the mechanisms underlying these dynamics.

3.1 Belief Updating

We first analyze the evolution of beliefs on and off the equilibrium path. Observe that the agent's deviation is never detected because of the full-support assumption for the feedback. Thus, any public history h^t is on the equilibrium path, and hence, the posterior belief is pinned down by Bayes' rule. Then, the Gaussian information structure of the game implies that all posteriors are also Gaussian; thus, any posterior belief is characterized by its mean and precision (the inverse of the variance).

Define the *public belief* as the common posterior belief under the expectation that the agents follow the equilibrium strategy profile. Formally, let $a^* = (a^*_1, \dots, a^*_N)$ be an equilibrium strategy profile, and given a public history $h^t = \{y_k\}_{k=0}^{t-1}$, define $\bar{h}^t_i = \{(\bar{a}_{ik}, y_k)\}_{k=0}^{t-1}$ recursively as $\bar{a}_{i0} = a^*_i(\emptyset)$ and $\bar{a}_{it} = a^*_i(\bar{h}^t_i)$. Note that $\bar{h}^t_i$ is a private history of agent i in which he follows the equilibrium strategy *for all* $t' < t$. Then, given the expectation of "no-deviation," an element of the feedback y_t that is purely informative about the state θ is

$$z_t \equiv y_t - \Delta\kappa_a \sum_i a_i^*(\bar{h}_i^t).$$

Note that if the agents do follow the equilibrium strategy for all $t' < t$, the signal z_t follows a normal distribution with mean $\Delta\kappa_\theta\theta$ and variance Δ/η_ε.

In each period, the public belief is updated based on z_t. Let μ_t and ν_t be the mean and the precision of the public belief in period t. Then, by standard Gaussian updating, μ_t and ν_t are recursively determined by

$$\mu_t = \frac{\nu_{t-1}\mu_{t-1} + \kappa_\theta\eta_\varepsilon z_{t-1}}{\nu_{t-1} + \Delta\kappa_\theta^2\eta_\varepsilon}, \quad \text{and} \quad \nu_t = \nu_{t-1} + \Delta\kappa_\theta^2\eta_\varepsilon. \tag{1}$$

The *private belief* of agent i does not necessarily follow the public belief, as he privately knows his effort level. Specifically, agent i updates his private belief based on the signal

$$\hat{z}_{it} \equiv y_t - \Delta\kappa_a \left(a_{it} + \sum_{j \neq i} a_j^*(\bar{h}_j^t) \right).$$

Then, the mean $\hat{\mu}_{it}$ and precision $\hat{\nu}_{it}$ of the private belief in period t are recursively determined by

$$\hat{\mu}_{it} = \frac{\nu_{t-1}\hat{\mu}_{it-1} + \kappa_\theta\eta_\varepsilon\hat{z}_{it-1}}{\nu_{t-1} + \Delta\kappa_\theta^2\eta_\varepsilon}, \quad \text{and} \quad \hat{\nu}_{it} = \nu_t. \tag{2}$$

Note that $\hat{\mu}_{it} = \mu_t$ as long as agent i follows the equilibrium strategy. In contrast, once an agent deviates from equilibrium effort choice, his private belief and the public belief diverge. For example, suppose that agent i deviates in period t and plays $a_{it} = a_i^*(\bar{h}_i^t) + \alpha$ for some $\alpha > 0$ and thereafter follows the strategy that the other agents anticipate (that is, he plays $a_i^*(\bar{h}_i^s)$ for any $s = t+1, \ldots, T$).[13] Then, for all future periods, the public belief is more optimistic than

[13] Clearly, playing $a_i^*(\bar{h}_i^s)$ need not be optimal for agent i following a deviation, as his belief off the equilibrium path may be distinct from the public belief. In Subsection 3.2, we provide a detailed discussion of the off-equilibrium behavior in the unique PBE.

agent i's private belief. In particular, for any $s > t$,

$$\hat{\mu}_{is} = \mu_s - \rho_s \alpha,$$

where

$$\rho_s = \left(\prod_{\tau=t+2}^{s} \frac{\partial \mu_{\tau+1}}{\partial \mu_\tau} \right) \cdot \frac{\partial \mu_{t+1}}{\partial z_t} \cdot \frac{\partial z_t}{\partial a_{it}} = \frac{\Delta \kappa_a \kappa_\theta \eta_\varepsilon}{\nu_s}$$

is the rate at which the deviation in period $t < s$ affects the belief divergence.[14]

Note that agent i's deviation does not bias his own belief about the state since he discounts the feedback according to his actual effort level. However, agent i's deviation biases the public belief, which discounts the observed feedback through the equilibrium action. Specifically, by devoting greater effort, each agent can increase the mean of the public belief μ_s (for any realization of noise ε_t) at a rate of ρ_s. This is precisely the mechanism that leads each agent to have additional incentives to increase his effort.

Finally, note that the precision of the posterior belief is deterministic and independent of any history. Since the speed of learning is independent of the action, the agents in our model do not have incentives for experimentation in choosing their optimal effort levels. This property greatly simplifies our equilibrium analysis, as becomes clear in the next subsection.

3.2 Equilibrium

We first state our main result.

Proposition 2. *There exists a unique perfect Bayesian equilibrium. In equilibrium, agent i's period-t action is*

$$a_i^*(h_i^t) = \xi_{it} \hat{\mu}_{it}, \tag{3}$$

where $\xi_{iT} = s_i / c_i$, *and*

$$\xi_{it} = \frac{s_i}{c_i} \left[1 + \sum_{k=t+1}^{T} e^{-r(k-t)\Delta} \sum_{j \neq i} \xi_{jk} \rho_k \prod_{l=t+1}^{k-1} (1 - \xi_{il} \rho_l) \right], \tag{4}$$

[14] We define $\prod_{\tau=t+2}^{s} \frac{\partial \mu_{\tau+1}}{\partial \mu_\tau} = 1$ for $s = t+1$.

for $t = 0, \ldots, T-1$.

The unique PBE of our model has a remarkably simple structure: *After any history*, the equilibrium action of each agent is linear in the mean of his private posterior belief.[15] We call the coefficient ξ_{it} agent i's *belief sensitivity of effort* in period t: It captures the rate at which the agent responds to changes in his belief ($\hat{\mu}_{it}$). Note that ξ_{it} is deterministic and varies only with the calendar time t. If the agents are homogeneous (that is, $c_i = c$ and $s_i = 1/N$ for all i), then ξ_{it}s are identical across agents and the unique PBE becomes symmetric. However, the agents may choose different actions off the equilibrium path, as their beliefs could diverge.

In the Appendix, we formally prove Proposition 2. Here, we provide an intuitive explanation. First, note that since we assume a quadratic cost function $c_i a^2/2$, agent i's optimal effort level equals his expected marginal benefit divided by c_i. Rewriting equations (3) and (4), we express the marginal benefit of effort as follows:

$$c_i a_{it}^* = \underbrace{s_i \hat{\mu}_{it}}_{\text{myopic benefit}} + \underbrace{\underbrace{e^{-r\Delta} s_i \hat{\mu}_{it} \sum_{j \neq i} \xi_{j,t+1} \rho_{t+1}}_{\text{effect on period } t+1} + \underbrace{e^{-2r\Delta} s_i \hat{\mu}_{it} \sum_{j \neq i} \xi_{j,t+2} \rho_{t+2} (1 - \xi_{i,t+1} \rho_{t+1})}_{\text{effect on period } t+2} + \cdots}_{\text{signal-jamming}}. \tag{5}$$

The first term in (5) captures the (direct) myopic benefit of effort, which is equal to the expected social benefit ($\hat{\mu}_{it}$) times agent i's share (s_i). The rest of the right-hand side captures the (indirect) benefit from manipulating the others' future beliefs. Specifically, this term captures the extent to which (agent i's share of) expected output increases as a result of an increase in i's effort in period t followed by optimal effort choices based on his private belief.

To understand the benefit from the signal-jamming effect, consider an upward deviation by agent i in period t in which he chooses $a_{it} = a_{it}^* + \alpha$, with $\alpha > 0$. For any realization of ε_t, such a deviation increases the mean of the period-$(t+1)$ public belief (μ_{t+1}) by $\rho_{t+1}\alpha$. Then, in the next period, each agent $j \neq i$ would increase his effort by $\xi_{j,t+1}\rho_{t+1}\alpha$ (recall that ξ_{jt} is the response rate of agent j's effort to a change in the posterior mean he holds in period t). Therefore, agent i's expected benefit from this increase in others' effort is $e^{-r\Delta} s_i \hat{\mu}_{it} \cdot$

[15] This result can be explained as follows: Due to the Gaussian signal structure, marginal impact of increased effort on belief is independent of the level of others' beliefs. Moreover, the marginal payoff gain from the belief divergence is constant due to the quadratic cost function.

$(\sum_{j\neq i}\xi_{j,t+1}\rho_{t+1}\alpha)$. Since the benefit from deviation is linear in α, the marginal benefit is constant and equal to the second term of (5).

For period $t+2$ (and thereafter), the effect of a deviation in period t becomes more complicated because the agents' beliefs diverge off the equilibrium path. If agent i plays $a_{it} = a^*_{it}+\alpha$ in period t, his private belief in period $t+1$ becomes more pessimistic than the public belief: $\hat{\mu}_{i,t+1} = \mu_{t+1} - \rho_{t+1}\alpha$. Then, in period $t+1$, the equilibrium effort level of agent i is smaller than what the others anticipate:

$$a^*_{i,t+1} = \xi_{i,t+1}\hat{\mu}_{i,t+1} = \xi_{i,t+1}(\mu_{t+1} \underbrace{-\rho_{t+1}\alpha}_{\text{divergence}}).$$

This divergence of agent i's effort makes the public belief in period $t+2$ downward biased. In particular, μ_{t+2} would decrease by $\rho_{t+2}\cdot\xi_{i,t+1}\rho_{t+1}\alpha$ relative to what its level would have been had the agent taken the anticipated action. Consequently, the belief divergence in period $t+1$ negatively affects agent i's incentive in period t. We refer to this negative incentive as the *ratcheting effect,* as the agent's current incentive to work is affected by the other agents' expectations in the future.[16]

As a result, the agent's marginal benefit of effort consists of the (positive) direct signal-jamming effect and the (negative) ratcheting effect. Note that similar to its effect on period $t+1$, agent i's deviation in period t directly increases μ_{t+2} by $\rho_{t+2}\alpha$. Therefore, agent i's deviation in period t, followed by the corresponding equilibrium strategy in period $t+1$, has a net effect of

$$\left[\underbrace{\xi_{j,t+2}\rho_{t+2}}_{\text{direct signal-jamming}} - \underbrace{\xi_{j,t+2}\rho_{t+2}\xi_{i,t+1}\rho_{t+1}}_{\text{ratcheting}}\right]\alpha = \xi_{j,t+2}\rho_{t+2}(1-\xi_{i,t+1}\rho_{t+1})\alpha$$

on agent j's period-$(t+2)$ effort choice. Summing over all agents $j\neq i$, multiplying by $s_i\hat{\mu}_{it}$

[16]The ratcheting effect, the effect of potentially causing high future expectations on the agent's current incentives, is extensively analyzed in the literature on dynamic agency models with asymmetric information (Weitzman, 1980; Freixas et al., 1985) and dynamic moral hazard with learning and symmetric uncertainty (Bhaskar, 2014; Prat and Jovanovic, 2014; Cisternas, 2016b; Bhaskar and Mailath, 2016).

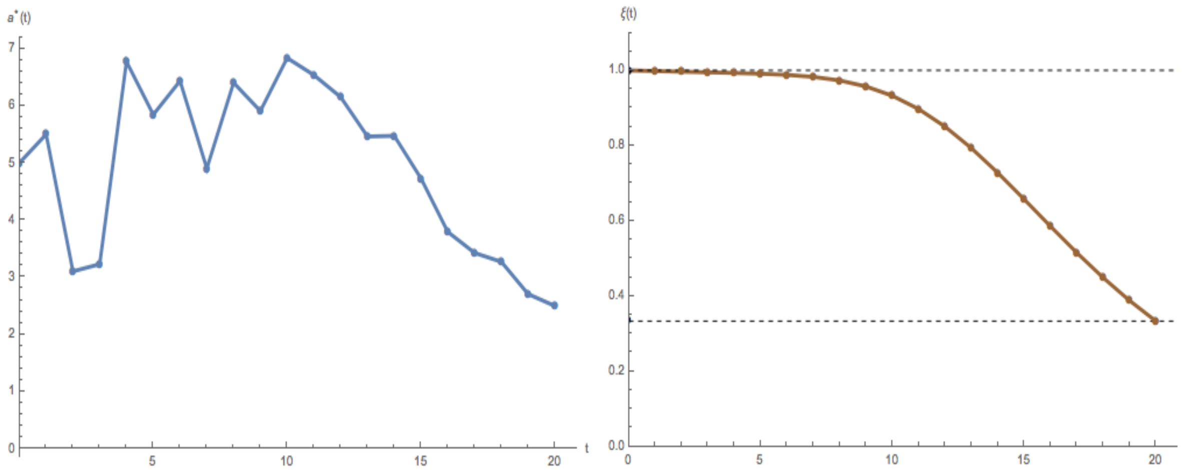
and discounting yields the coefficient (of α) equal to the third term of (5). Iterating this reasoning yields the expression in Proposition 2.

Remark 3. The direct signal-jamming effect in period t is linear in each $\xi_{jk}\,(k > t)$. By devoting greater effort, each agent can directly change the future posterior, to which the other agents respond in a linear way. However, the ratcheting effect is (at least) quadratic in ξ_{ik} and ξ_{jk}: A deviation creates a belief divergence in future periods, and the resulting divergence in the expected effort level in turn leads to the belief distortion in periods further in the future. The implication of this difference becomes more evident when we consider the continuous-time limit of the equilibrium below.

The uniqueness result is proven by a backward induction argument. Note that the above argument for the marginal benefit holds after any history, regardless of whether an agent has previously deviated. In the final period ($t = T$), after any history h_i^T, each agent has a unique best response $a_i^*(h_i^T) = (s_i/c_i)\hat{\mu}_{iT}$, which is linear in the mean of the private belief. Now suppose that for some t, the equilibrium strategy after any history h_i^k is linear in $\hat{\mu}_{ik}$ for all $k = t+1,\ldots,T$. Then, each agent's best response in period t is unique since the cost of effort is convex while the benefit is linear. Furthermore, our linear-quadratic-Gaussian structure implies that the unique best response is also linear in $\hat{\mu}_{it}$. In the Appendix, we present a formal proof based on this argument.

Equilibrium dynamics Figure 1 describes the dynamics of the unique PBE when the agents are homogeneous ($c_i = 1$ and $s_i = 1/N$ for all i). Note that with homogeneous agents, the unique PBE is symmetric; i.e., $\xi_{it} = \xi_t$ for all i. The left panel shows a realization of the equilibrium effort on the equilibrium path (where $\hat{\mu}_{it} = \mu_t$). The equilibrium effort level $a_{it}^* = \xi_t \mu_t$ is stochastic and typically nonmonotonic over time. This is because the dynamics of the posterior mean μ_t depend on the realized feedback. However, the coefficient of the equilibrium action (belief sensitivity of effort) is deterministic and has more consistent properties. In what follows, we analyze the equilibrium properties by mainly focusing on the dynamics of the belief sensitivity.

The dynamics of a symmetric ξ_t over time are depicted in the right panel of Figure 1.



(a) a realization of the equilibrium effort a^*_{it} (b) belief sensitivity ξ_t

Figure 1: Dynamics of the unique PBE with the homogeneous agents $(N = 3, T = 20)$

Recall that the myopically optimal level of belief sensitivity—that is, the level without the signal-jamming effect—is $s_i/c_i = 1/N$ (lower dashed line), while the socially efficient level is $1/c_i = 1$ (upper dashed line). In the graph, the equilibrium ξ_t decreases over time and lies between the two dashed lines.

The intuition for decreasing belief sensitivity is twofold. First, as t increases, there would be fewer remaining periods during which the coworkers make effort choices, and thus, the agents' return to influencing the others' beliefs declines. Second, as the agents learn θ more precisely over time, they place a smaller weight on the new feedback in updating their beliefs, making it more difficult to affect this belief by changing the effort level.

Although the above intuition suggests that the equilibrium ξ_{it} should generally be monotonic, this is not always the case. Nonmonotonicity may result when the ratcheting effect dominates the direct signal-jamming effect. Such a phenomenon may occur when the belief sensitivity in the next period is large. For example, suppose that the (homogeneous) agents in period t expect a very large (symmetric) ξ_{t+1} (where $t+1 < T$). Then, contributing more effort in period t creates a very large divergence in expectations of $a_{i,t+1}$ between agent i and the other agents, which in turn downward biases the period-$(t+2)$ public belief by a large amount. This quadratic ratcheting effect may dominate the direct signal-jamming effect, and thus, ξ_t may be lower than ξ_{t+1}. In the Online Appendix, we discuss this issue in further detail.

Such nonmonotonicity disappears when the "real-time" length of a period becomes shorter. Note that as the period length becomes shorter, the agents receive more frequent feedback and can frequently adjust their effort levels. In this case, as we solve the equilibrium by backward induction, the equilibrium ξ_{it} cannot exhibit large "sudden jumps". Therefore, although the ratcheting effect strengthens gradually (as t goes backward), it does not dominate the signal-jamming effect, leading to monotonic dynamics.

In the next subsection, we consider the continuous-time limit of the model where the period length becomes arbitrarily small. In this limiting environment, we establish the monotonicity of ξ_{it} and conduct comparative statics.

Role of Imperfect Monitoring Before we further analyze the characteristics of the unique PBE, let us emphasize the importance of our assumption that a_{it} is unobservable. The following proposition shows that if the agent's effort level is observable to others, then there is no scope for signal-jamming and each agent chooses the myopically optimal effort level. Its proof is straightforward and thus omitted.

Proposition 3. *In the perfect monitoring case, there exists a unique perfect Bayesian equilibrium where for any $t = 0, \ldots, T$,*

$$a_{it}^* = \frac{s_i}{c_i}\mu_t.$$

Interestingly, the classic literature on team production has regarded the inability (or a high cost) to monitor individual effort as a major obstacle to inducing cooperation. As Alchian and Demsetz (1972) write,

> ...In team production, marginal products of cooperative team members are not so directly and separably (i.e., cheaply) observable. What a team offers to the market can be taken as the marginal product of the team but not of the team members. The costs of metering or ascertaining the marginal products of the team's members is what calls forth new organizations and procedures.

From the perspective of team design, Proposition 3 implies that in environments in which monitoring is costly, it may be preferable to choose a project with uncertain prospects instead of investing in the monitoring structure and establishing formal contracts.[17]

3.3 Continuous-Time Limit and Comparative Statics

In this subsection, we consider a continuous-time limit in which the feedback (and the corresponding effort adjustment) is arbitrarily frequent. Specifically, we fix the real-time length $\tau = T\Delta$ of the game and consider the limit of equilibrium behavior as $\Delta \to 0$.

3.3.1 Equilibrium under Continuous-Time Limit

To derive the continuous-time limit of the equilibrium, we first describe the equilibrium strategies in recursive form. Define $S_{ij,t} (t = 0, \ldots, T)$ recursively as

$$S_{ij,t} = \xi_{j,t+1}\rho_{t+1} + e^{-r\Delta}(1 - \xi_{i,t+1}\rho_{t+1})S_{ij,t+1}, \tag{6}$$

with $S_{ij,T} = 0$. The variable $S_{ij,t}$ captures the effect of agent i's signal-jamming in period t, resulting from changes in agent j's effort level. Then, (4) can be rewritten as

$$\xi_{it} = \frac{s_i}{c_i}\left[1 + e^{-r\Delta}\sum_{j \neq i} S_{ij,t}\right]. \tag{7}$$

Summing (6) over $j \neq i$ and substituting for $\sum_{j \neq i} S_{ij,t}$ and $\sum_{j \neq i} S_{ij,t+1}$ from (7) yields a recursive formulation for ξ_{it}:

$$\xi_{it} = \frac{s_i}{c_i} + e^{-r\Delta}\left[\frac{s_i}{c_i}\sum_{j \neq i}\xi_{j,t+1}\rho_{t+1} + (1 - \xi_{i,t+1}\rho_{t+1})\left(\xi_{i,t+1} - \frac{s_i}{c_i}\right)\right]. \tag{8}$$

Then, writing the variables in terms of Δ, re-arranging, and taking $\Delta \to 0$, we obtain the following system of differential equations: For $i = 1, \ldots, N$,

[17] Bonatti and Hörner (2011) also show that perfect monitoring may lead to more inefficient outcomes. Nevertheless, the mechanism underlying their result differs from that in our paper. In Bonatti and Hörner (2011), when it is *observed* that an agent worked hard, the agents are incentivized against contributing effort in the future. In contrast, in our paper, the agent's effort shows strategic complementarity over time *when effort is unobservable*, which provides positive incentives under imperfect monitoring.

$$\dot{\xi}_i(t) = \underbrace{r\left(\xi_i(t) - \frac{s_i}{c_i}\right)}_{\text{discounting}} - \underbrace{\frac{\eta_\varepsilon \kappa_\theta \kappa_a}{\nu_0 + \eta_\varepsilon t \kappa_\theta^2}\left(\frac{s_i}{c_i}\sum_{j=1}^{N}\xi_j(t) - \xi_i(t)^2\right)}_{\text{signal-jamming}}. \tag{9}$$

Together with the terminal conditions $\xi_i(\tau) = s_i/c_i$, the above system of differential equations fully describes the equilibrium dynamics over time.[18,19] In the Online Appendix, we formulate the continuous-time counterpart of the main model and show that (i) the equilibrium of the continuous-time model is described by (9) and (ii) the equilibrium of the discrete-time model (weakly) converges to the continuous-time equilibrium as $\Delta \to 0$.

The differential equation (9) has a simple and intuitive form. The first term captures the effect of discounting. Note that whenever $\xi_i(t)$ is above the myopically optimal level (s_i/c_i), the first term is positive, and thus, the discounting effect decreases the incentive in the earlier phases (bear in mind that we compute $\xi_i(t)$ backwards from $t = \tau$). The second term captures the effect of signal-jamming, and its coefficient is a function of the information parameters ($\nu_0, \eta, \kappa_\theta$ and κ_a). It consists of a linear component and a quadratic component, which capture the direct signal-jamming effect and the ratcheting effect, respectively.

Figure 2 illustrates the evolution of belief sensitivity in the limit, where $\Delta \to 0$, with homogeneous agents (left panel) and heterogeneous agents (right panel). It shows that the signal-jamming effect remains nontrivial in the continuous-time limit. In what follows, we use the simple form of (9) to further analyze the properties of the equilibrium.

3.3.2 Equilibrium Properties

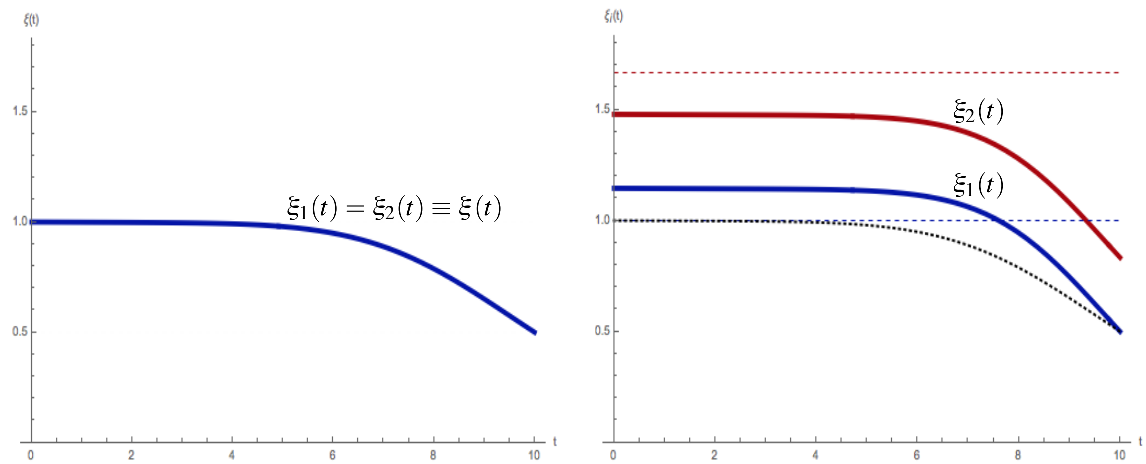
We begin by establishing the monotonicity of equilibrium belief sensitivity $\xi_i(t)$.

Proposition 4. *For any i, $\xi_i(t)$ is monotonic and decreasing over time. Moreover, for any $t \in [0, \tau]$, $\xi_i(t) \in [\underline{\xi}_i, \bar{\xi}_i)$, where*

$$\underline{\xi}_i = \frac{s_i}{c_i}, \qquad \bar{\xi}_i = \sqrt{\frac{s_i}{c_i}}\sum_{j=1}^{N}\sqrt{\frac{s_j}{c_j}}.$$

[18] In this subsection, we slightly abuse notation and refer t as the real-time in the game.

[19] This system is a backward Riccati equation. In the proof of Proposition 4, we show the existence and uniqueness of the solution. When s_i and c_i are the same for all i, the system has a closed-form solution expressed by confluent Hypergeometric and Laguerrel functions.



(a) homogeneous agents ($c_1 = c_2 = 1$) (b) heterogeneous agents ($c_1 = 1, c_2 = 0.6$)

Figure 2: Equilibrium $\xi_i(t)$ under the continuous-time limit $(N = 2, \tau = 10, s_i = 1/2)$

The intuition for a monotonically decreasing $\xi_i(t)$ is provided in the previous subsection. In addition, the proposition also establishes the lower and upper bounds on the equilibrium $\xi_i(t)$. The lower bound $\underline{\xi}_i$ is the myopically optimal level that would be attained in the absence of signal-jamming incentives.

The existence of the upper bound $\bar{\xi}_i$ is due to the ratcheting effect, which appears in the quadratic term of (9). Suppose that we solve (9) backwards from the terminal point $t = \tau$. At $t = \tau$, as $\xi_i(\tau) = s_i/c_i$, the linear term $(\frac{s_i}{c_i} \sum_{j=1}^{N} \xi_j(t))$ is greater then the quadratic term $(\xi_i(t)^2)$, and thus, the signal-jamming incentive becomes greater as t goes backward. However, as $\xi_i(t)$ becomes larger, the quadratic term catches up to the linear term, which prevents the belief sensitivity from being greater than $\bar{\xi}_i$.[20]

The next proposition states the equilibrium properties with respect to the cost parameter and the share structure. Its proof is straightforward from equation (9) and Proposition 4 and thus is omitted.

Proposition 5. *Consider the continuous-time limit of the unique PBE.*

1. *For any $t \in [0, T]$, $\xi_i(t)$ decreases in c_i.*

[20] In contrast, in Holmström (1999)'s career concerns model, the ratcheting effect does not preclude equilibrium action from diverging under limits (e.g., $\kappa_a \to \infty$). This is because in our model, the return to jamming the feedback signal is endogenous and based on others' belief sensitivity, which is also subject to the ratcheting effect. The reduction in others' belief sensitivity due to the ratcheting effect compounds the negative impact on each agent's effort choice, eventually bounding it.

2. *For any $t \in [0,T)$, $\xi_i(t)$ decreases in $c_j (j \neq i)$.*

3. *Suppose that the share structure $s^* = (s_1^*, \ldots, s_N^*)$ is set by $s_i^* \equiv \frac{\frac{1}{c_i}}{\sum_{j=1}^{N} \frac{1}{c_j}}$. Then, $\bar{\xi}_i = 1/c_i$ for all $i = 1, \ldots, N$.*

The intuition for part 1 is straightforward: The agent contributes less effort when his marginal cost increases. Perhaps more interestingly, part 2 states that agent i's effort level decreases in the marginal cost of other agents. This is because the agent's marginal benefit of effort is increasing in the other agents' belief sensitivity, which is decreasing in their own cost. This effect is illustrated in Figure 2. The left panel describes the symmetric $\xi(t)$ of a homogeneous two-person team, and the right panel shows $\xi_1(t)$ and $\xi_2(t)$ when agent 2's marginal cost has decreased. Note that *both* $\xi_1(t)$ and $\xi_2(t)$ lie above the symmetric $\xi(t)$ (black dotted line in the right panel): If one agent's cost is reduced, then both agents would choose higher effort.

Part 3 shows that there exists a sharing rule that makes the upper bound on agent i's belief sensitivity ($\bar{\xi}_i$) coincide with the socially efficient level ($1/c_i$). Figure 2 (right panel) shows that generally $\bar{\xi}_i$ does not coincide with $1/c_i$ (depicted as dashed lines of the respective color): $\xi_1(t)$ initially lies above the socially efficient level, while agent 2 always underinvests in his effort. If an agent's cost is higher than that of other agents, his signal-jamming incentives may be inefficiently strong, as the other agents are more responsive to belief changes.

The next proposition establishes the comparative statics result for the discount rate and the information parameters.

Proposition 6. *In the unique PBE of the model, for any $t \in [0, \tau)$,*

1. *$\xi_i(t)$ decreases in r.*

2. *$\xi_i(t)$ decreases in ν_0 and increases in η_ε.*

3. *$\xi_i(t)$ increases in κ_a but is nonmonotone in κ_θ.*

That $\xi_i(t)$ decreases in r is intuitive: A larger r makes the future less important and thus decreases the signal-jamming incentive. The intuition for parts (ii) and (iii) can be explained by the coefficient $\left(\frac{\eta_\varepsilon \kappa_\theta \kappa_a}{\nu_0 + \eta_\varepsilon t \kappa_\theta^2}\right)$ of the second term of (9): This coefficient becomes larger when

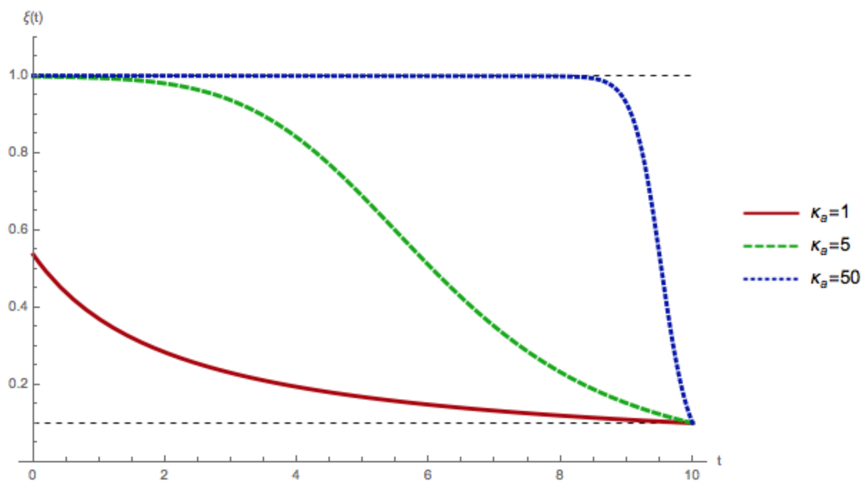


Figure 3: Equilibrium $\xi(t)$ in the homogeneous agents case, with different values of κ_a

future beliefs become more sensitive to variations in current effort, and consequently, the marginal benefit of current effort increases. This, in turn, happens when the impact of effort on feedback (κ_a) increases or when future beliefs become more sensitive to feedback either due to a decrease in initial precision (ν_0) or an increase in the signal precision (η). The effect of κ_θ on the signal-jamming effect is nonmonotonic. Specifically, signal-jamming incentives disappear when κ_θ is too low (feedback contains almost no information about θ) or too high (feedback is extremely precise). This implies that there is an interior value of κ_θ that maximizes the belief sensitivity of effort.

The next proposition establishes the effect of team size, and it states that as the individual effort level decreases in N, the total effort level increases.

Proposition 7. *If the agents are homogeneous ($c_i = c$ and $s_i = 1/N$ for all i), then for any $t \in [0,T]$, $\xi(t)$ decreases in N, and for any $t \in [0,T)$, $N\xi(t)$ increases in N.*

3.3.3 Convergence to the Socially Efficient Level

Finally, the following proposition shows that with the share structure established in Proposition 5, the equilibrium $\xi_i(t)$ can be arbitrarily close to the socially efficient level in certain limiting cases.

Proposition 8. *Suppose that the share structure $s^* = (s_1^*, \ldots, s_N^*)$ is set as*

$$s_i^* \equiv \frac{\frac{1}{c_i}}{\sum_{j=1}^N \frac{1}{c_j}}.$$

Then, as $\kappa_a \to \infty$, the agents' equilibrium belief sensitivity $\xi_i(t)$ for any $t \in [0, \tau)$ converges (pointwise) to $1/c_i$.

Figure 3 shows the equilibrium $\xi(t)$ in the homogeneous agents case with different values of κ_a ($\kappa_a = 1, 5, 50$). Note that if the agents have the same c_i, then $s_i^* = 1/N$, and thus, the equal share structure induces the socially efficient level. Note that as κ_a increases, the equilibrium $\xi(t)$ becomes arbitrarily close to the socially efficient level ($1/c = 1$) for almost the full length of the horizon.

4 Stationary Model

In this section, we consider an infinite-horizon version of our model in which the state θ evolves stochastically over time. In doing so, we establish that the our main mechanism continues to be present in the long run as long as the uncertainty over the state is never fully resolved. Moreover, the tractability of the stationary model allows us to address additional considerations such as heterogeneity of agents with respect to their information about the state. We undertake the latter exercise in Section 4.1.

For ease of exposition, we consider the case in which $c_i = 1$ and $s_i = 1/N$ for all i. Further, we reinterpret the discount factor δ as the probability of project survival: Partnership ends with probability $1-\delta$ in each period, and each agent receives his share of the output at the end of the partnership.

Let θ_t be the state of the world in period t. We assume that θ_t follows a random walk

$$\theta_{t+1} = \theta_t + \sigma_t,$$

where σ_t is independently and identically drawn from the distribution $\mathcal{N}(0, \Delta/\nu_\sigma)$. Similar to the finite-horizon model, the period-t feedback is given by $y_t = \Delta\left[\kappa_\theta \theta_t + \kappa_a \sum_{i=1}^N a_{it} + \varepsilon_t\right]$, where $\kappa_\theta, \kappa_a > 0$ are constants and $\varepsilon_t \sim \mathcal{N}(0, 1/\nu_\varepsilon)$. Assume that σ_t and ε_t are independent over time and are of one another.

As in our main model, the posterior belief about the state after any history follows a normal distribution. Let μ_t and ν_t be the mean and precision of the public belief about θ_t at the beginning of period t, and let μ_t' and ν_t' describe the public belief *after* the feedback y_t is realized. Then, we have $\mu_t' = \frac{\nu_t \mu_t + \kappa_\theta \eta_\varepsilon z_t}{\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon}$ and $\nu_t' = \nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon$, where $z_t \equiv y_t - \Delta \kappa_a \sum_i a_i^*(\bar{h}_i^t)$ is the signal that the agents use to update the public belief given the equilibrium strategy a_i^* and the corresponding "no-deviation" history $\bar{h}_i^t$ (which is defined in Subsection 3.1).

Taking into account the effect of σ_t, the period-$(t+1)$ public belief is characterized by

$$\mu_{t+1} = \mu_t' = \frac{\nu_t \mu_t + \kappa_\theta \eta_\varepsilon z_t}{\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon}, \tag{10}$$

$$\nu_{t+1} = \left(\frac{1}{\nu_t'} + \frac{1}{\nu_\sigma}\right)^{-1} = \frac{\left(\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon\right) \nu_\sigma}{\Delta \left(\nu_t + \Delta \kappa_\theta^2 \eta_\varepsilon\right) + \nu_\sigma}. \tag{11}$$

The private belief of agent i is updated using the same Gaussian updating process, but with the signal $\hat{z}_{it} = y_t - \Delta \kappa_a (a_{it} + \sum_{j \neq i} a_j^*(\bar{h}_j^t))$ instead of z_t.

Observe that, as in our main model, the belief precision ν_t is deterministic and independent of the strategy profile. Moreover, it is easy to show that for any value of the initial precision ν_0, as t goes to infinity, ν_t converges to a stationary level ν^*, where ν^* is given by[21]

$$\nu^* = \frac{\eta_\varepsilon \kappa_\theta^2}{2} \left(-\Delta + \sqrt{\Delta^2 + \frac{4\nu_\sigma}{\eta_\varepsilon \kappa_\theta^2}}\right). \tag{12}$$

We are interested in constructing a Markov perfect equilibrium with the same structure as the unique PBE of our main model:[22] In the equilibrium, the agent's action is linear in the mean of his private belief, that is, $a_i^*(h_i^t) = \xi_t \hat{\mu}_{it}$.[23] We construct the linear Markov perfect

[21] The stationary level of ν^* is derived by setting $h_t = \nu_{t+1} = \nu^*$ in equation (11):

$$\nu^*(\Delta \left(\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2\right) + \nu_\sigma) = (\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2) \nu_\sigma.$$

This quadratic equation has a unique positive solution, as shown above.

[22] By a Markov perfect equilibrium, we mean a PBE in which the agents' equilibrium strategies after any history depend only on their private beliefs.

[23] Recall that while the belief sensitivity is stationary, a realization of the equilibrium action is stochastic and typically nonmonotonic over time.

equilibrium for any value of initial precision $\nu_0 > 0$ (Proposition 9). However, for heuristic purposes, let us consider an environment in which $\nu_0 = \nu^*$, meaning that the precision is stationary over time.

Under stationary precision, there exists a Markov perfect equilibrium with a stationary sensitivity level, that is, $\xi_t = \xi^*$ for all t. To compute ξ^*, let us consider the effect of a deviation in period t on the future beliefs in this equilibrium. Suppose that agent i deviates to $a = \xi^* \hat{a}_{it} + \alpha$. Then, the period-$(t+1)$ public posterior mean is given by $\mu_{t+1} = \hat{\mu}_{i,t+1} + \Lambda_a \alpha$, where $\Lambda_a \equiv \frac{\partial \mu_{t+1}}{\partial a_t} = \frac{\Delta \eta_\varepsilon \kappa_\theta \kappa_a}{\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2}$. From period $(t+2)$ onward, the public belief and agent i's private belief diverge, which influences the return on the deviation in period t as follows: While the other agents expect agent i to play $\xi^* \mu_{t+1}$, agent i actually plays $\xi^* \hat{\mu}_{i,t+1} = \xi^* \mu_{t+1} - \xi^* \Lambda_a \alpha$. Therefore, the period-$(t+2)$ public posterior mean is given by

$$\mu_{t+2} = \hat{\mu}_{i,t+2} + \Lambda_a (\Lambda_\mu - \xi^* \Lambda_a) \alpha,$$

where $\Lambda_\mu \equiv \frac{\partial \mu_{t+1}}{\partial \mu_t} = \frac{\nu^*}{\nu^* + \Delta \eta_\varepsilon \kappa_\theta^2}$. By iteration, we have $\mu_{t+k} = \hat{\mu}_{i,t+k} + \Lambda_a (\Lambda_\mu - \xi^* \Lambda_a)^{k-1} \alpha$ for any $k \geq 2$.

Then, the optimal effort level, which equals the marginal benefit of effort, is given by

$$a_{it}^* = \frac{\hat{\mu}_{it}}{N} \left(1 + (N-1) \xi^* \sum_{k=1}^{\infty} e^{-r\Delta k} \Lambda_a (\Lambda_\mu - \xi^* \Lambda_a)^{k-1} \right).$$

Therefore, we have

$$\xi^* = \frac{1}{N} \left(1 + (N-1) \frac{e^{-r\Delta} \xi^* \Lambda_a}{1 - e^{-r\Delta} (\Lambda_\mu - \xi^* \Lambda_a)} \right),$$

which yields a quadratic equation for ξ^*. Simplifying, we have

$$N(\xi^*)^2 - N(1-\Gamma)\xi^* - \Gamma = 0, \tag{13}$$

where $\Gamma = \frac{1 - e^{-r\Delta} \Lambda_\mu}{e^{-r\Delta} \Lambda_a} = \frac{(1 - e^{-r\Delta}) \nu^* + \Delta \eta_\varepsilon \kappa_\theta^2}{e^{-r\Delta} \Delta \eta_\varepsilon \kappa_\theta \kappa_a}$. There exists a unique positive solution for ξ^*, which is given by

$$\xi^* = \frac{1 - \Gamma + \sqrt{(1-\Gamma)^2 + 4\Gamma/N}}{2}. \tag{14}$$

The next proposition states the general result that for any initial precision ν_0, under a sufficiently small Δ, there exists a linear Markov perfect equilibrium in which the belief sensitivity of effort converges to the stationary level.

Proposition 9. *Fix $\nu_0 > 0$. There exists $\bar{\Delta} > 0$ such that for any $\Delta < \bar{\Delta}$, there exists an initial value of ξ_0 such that the agent's Markovian strategy*

$$a^*_{it}(\hat{\mu}_{it}) = \xi_t \hat{\mu}_{it},$$

where (ν_t, ξ_t), is recursively defined as

$$\begin{aligned}
\nu_{t+1} &= \frac{\left(\nu_t + \Delta\kappa_\theta^2 \eta_\varepsilon\right)\nu_\sigma}{\Delta\left(\nu_t + \Delta\kappa_\theta^2\eta_\varepsilon\right) + \nu_\sigma}, \\
\xi_t &= \frac{1}{N} + e^{-r\Delta}\frac{\Delta\eta_\varepsilon\kappa_\theta\kappa_a}{\nu_t + \Delta\eta_\varepsilon\kappa_\theta^2}\left(\xi_{t+1} + \frac{\nu_{t+1}}{\Delta\eta_\varepsilon\kappa_\theta\kappa_a}\left(\xi_{t+1} - \frac{1}{N}\right) - \xi_{t+1}\xi_{t+1}\right),
\end{aligned}$$

is a Markov perfect equilibrium. Moreover, $(\nu_t, \xi_t) \to (\nu^*, \xi^*)$ as $t \to \infty$, where ν^* and ξ^* are given by (12) and (14), respectively.

In the Online Appendix, we describe the detailed construction of the linear Markov perfect equilibrium. Specifically, we construct an agent's dynamic programming problem where his value function (continuation payoff) is a function of the mean of the public belief μ_t and his private belief $\hat{\mu}_{it}$. Making use of our linear-quadratic-Gaussian framework, we guess that a quadratic value function the agent's optimal strategy is linear in $\hat{\mu}_{it}$ and the coefficients of the value function are recursively defined. Then, using the phase diagram, we show that for any ν_0, there exists a unique value of ξ_0 following which $\xi_t \to \xi^*$. Since it satisfies the transversality condition, the sequence $\{\xi_t\}$ describes an equilibrium strategy profile.

Comparative Statics Using the simple form of the stationary belief sensitivity ξ^* (equation (14)), we derive the following comparative statics result.

Proposition 10. *The following properties hold for ξ^*:*

1. *For any $N \geq 2$, $\xi^* \in (1/N, 1)$; ξ^* decreases in N.*

2. $\frac{\partial \xi^*}{\partial \delta} > 0$.

3. $\frac{\partial \xi^*}{\partial \kappa_\theta} < 0$.

4. $\frac{\partial \xi^*}{\partial \kappa_a} > 0$, *and* $\lim_{\kappa_a \to \infty} \xi^* = 1$.

Parts 2 and 4 are analogous to the results from our main model, and they share the same intuition (See Propositions 6 and 7). The other two points require further discussion. Part 1 states that the stationary ξ^* is between the myopically optimal level ($s_i/c_i = 1/N$) and the socially efficient level ($1/c_i = 1$). This is from the assumption that the agents are homogeneous and have an equal share structure. Similar to Section 3, if the agents are heterogeneous, then the stationary belief sensitivity of some may exceed the socially efficient level.

Part 3 shows that ξ^* is monotonically decreasing in κ_θ, which differs from the results in Section 3. Recall that in the main model, the impact of κ_θ on ξ_{it} is nonmonotone (Proposition 6): The signal-jamming incentive disappears when κ_θ is too low because the feedback becomes extremely uninformative *relative to* the prior belief about the persistent state θ. If the state is stochastic, however, as the feedback becomes uninformative about the state, the stationary precision ν^* also becomes smaller. In this case, the feedback actually becomes more informative relative to the existing information, leading to the monotonic relationship between κ_θ and ξ^*.

Impact of Flexibility The last proposition of Section 4 states that as the agents' actions become more flexible, the signal-jamming incentives become stronger.

Proposition 11. *In the stationary MPE of the model,* $\frac{\partial \xi^*}{\partial \Delta} < 0$.

The intuition for Proposition 11 is straightforward. Suppose that Δ is arbitrarily large, and thus, for a long time, agents are not able to adjust their effort levels to accord with the available information. Then, the effect of information on the future effort level would occur far in the future, and hence, the signal-jamming incentive vanishes and the equilibrium belief sensitivity becomes arbitrarily close to the myopically optimal level. However, as Δ becomes smaller, the agents have more frequent opportunities to manipulate others' beliefs, leading to a higher benefit of effort.

Proposition 11 highlights an interesting comparison between the signal-jamming mechanism in this paper and the standard punishment-based mechanism. In the standard repeated partnership model, it is well-known that with imperfect public monitoring, as $\Delta \to 0$, the effectiveness of the punishment scheme could be limited (Abreu et al., 1991) or cooperation may not be possible at all (Sannikov and Skrzypacz, 2007). Under flexible actions, the trigger strategy profile must punish the agents based on the noisy information, which increases the cost of type I error, which eventually outweighs the benefit from future cooperation. In contrast, our Markovian equilibrium shows that the signal-jamming incentives become even stronger as the production becomes flexible.

4.1 Asymmetric Information

In this section, we consider the effect of information asymmetry among the agents on team production. In particular, we extend the infinite-horizon, stochastic-state model to the case in which some of the team members perfectly observe the state θ_t at the beginning of each period.[24] We refer to such team members as *experts*. However, *novices* refer to the other team members who arrive with an initial prior, as in the previous section, and update their beliefs based on the feedback.[25] In what follows, the superscripts e and n denote variables for experts and novices, respectively.[26]

Fix a team of agents, and let $N^e, N^n (N^e + N^n = N)$ be the number of experts and novices. We continue to assume that the state θ_t follows a random walk process, that is, $\theta_{t+1} = \theta_t + \sigma_t$. We construct an equilibrium in Markovian strategies in which the expert's effort level is linear in the period-t state and the novice's effort level is linear in the mean of the period-t private belief, that is,

$$a_t^e = \gamma_t \theta_t, \qquad a_t^n = \xi_t \hat{\mu}_{it}.$$

Similar to the previous analysis, let ν_t be the precision of novices' period-t public belief. Then,

[24]In the Online Appendix, we analyze a continuous-time model under asymmetric information in finite horizon. In this model, the existence of a deadline allows us to reduce the problem to a boundary value problem.

[25]The experts in our model play a similar role to the "leader" in Hermalin (1998). Similar to Hermalin (1998), we show that the presence of experts does not lead to full information revelation (this subsection) and may improve welfare (Subsection 5.1).

[26]We assume that whether each agent is an expert or a novice is public information. An interesting extension would be the case in which there is asymmetric information about the type of each agent.

a sequence of a triplet $\{(\gamma_t, \xi_t, \nu_t)\}_{t=0}^{\infty}$ completely describes the equilibrium strategy profile.

Under asymmetric information, the experts' actions can affect the precision of novices' belief ν_t. In particular, given the above linear Markovian strategy profile, and given the belief that the novices have never deviated in the past history (which implies that $\hat{\mu}_{it} = \mu_t$ for all novices), the novices understand that the feedback y_t is of the following form

$$\begin{aligned} y_t &= \Delta\left[\kappa_\theta \theta_t + \kappa_a(N^e a_t^e + N^n a_t^n) + \varepsilon_t\right], \\ &= \Delta\left[m_t \theta_t + \kappa_a N^n \xi_t \mu_t + \varepsilon_t\right], \end{aligned}$$

where $m_t = \kappa_\theta + \kappa_a N^e \gamma_t$. Then, after observing y_t, the novices use the signal $z_t = y_t - \Delta\kappa_a N^n \xi_t \mu_t$ to update the public belief about the state, which is given by

$$\mu_{t+1} = \frac{\nu_t \mu_t + \eta_\varepsilon m_t z_t}{\nu_t + \Delta\eta_\varepsilon m_t^2}, \qquad \nu_{t+1} = \frac{(\nu_t + \Delta\eta_\varepsilon m_t^2)\nu_\sigma}{\Delta\left(\nu_t + \Delta\eta_\varepsilon m_t^2\right) + \nu_\sigma}. \tag{15}$$

Note that as the experts' response rate (γ_t) increases, the updated mean μ_{t+1} weighs more heavily toward z_t and the precision ν_{t+1} increases.

In the Online Appendix, we solve a dynamic program to derive the following recursive formula for γ_t and ξ_t:

$$\begin{aligned} \gamma_t &= \frac{1}{N} + e^{-r\Delta}\frac{\Delta\eta_\varepsilon m_t \kappa_a}{\nu_t + \Delta\eta_\varepsilon m_t m_t}\left(\frac{N^n}{N}\xi_{t+1} + \frac{\nu_{t+1}}{\Delta\eta_\varepsilon m_{t+1}\kappa_a}\left(\gamma_{t+1} - \frac{1}{N}\right)\right), \\ \xi_t &= \frac{1}{N} + e^{-r\Delta}\frac{\Delta\eta_\varepsilon m_t \kappa_a}{\nu_t + \Delta\eta_\varepsilon m_t m_t}\left(\frac{N^n}{N}\xi_{t+1} + \frac{\nu_{t+1}}{\Delta\eta_\varepsilon m_{t+1}\kappa_a}\left(\xi_{t+1} - \frac{1}{N}\right) - \xi_{t+1}^2\right). \end{aligned}$$

The next proposition shows that the above system, combined with the recursive equation for ν_t (equation (15)), has a stationary solution.

Proposition 12. *There exists a unique triplet (γ^*, ξ^*, ν^*) such that if $\nu_0 = \nu^*$, the linear Markovian strategies*

$$a_{it}^{e*}(\theta_t) = \gamma^* \theta_t, \qquad a_{it}^{n*}(\hat{\mu}_{it}) = \xi^* \hat{\mu}_{it},$$

constitute a Markov perfect equilibrium. In this equilibrium, $\nu_t = \nu^*$ after any history.[27]

[27] For a general value of ν_0, finding a sequence of (γ_t, ξ_t, ν_t) that converges to the stationary value is analytically

5 Application: Team Composition

Our model, and the unique PBE with a simple structure, can be used to analyze various questions related to team production and optimal team design. In this section, we consider optimal team composition when (i) the agents have different information about the state, and (ii) the agents are heterogeneous in their effort cost. Our statements in this section are based on numerical simulations or simple examples.

5.1 The Value of Experts

A particularly interesting dimension of heterogeneity concerns the agents' expertise in the project, as captured by the asymmetric information model in Section 4.1. In this subsection, we analyze the stationary equilibrium of the asymmetric information model to address the following question: What is the optimal number of experts in a fixed-sized team?

To measure the welfare effect of team composition, we calculate the ex ante payoff of the experts and the novices in the stationary equilibrium. In the stationary equilibrium, given any realization of θ_t, the public belief μ_t follows a normal distribution with mean θ_t and variance $1/\nu^*$. Then, the expert's payoff is given by

$$\mathbb{E}\left[\frac{\theta_t}{N}(N^e\gamma^*\theta_t + N^n\xi^*\mu_t) - \frac{\gamma^{*2}\theta_t^2}{2}\right] = \frac{N^e}{N}\gamma^*\theta_t^2 + \frac{N^n}{N}\xi^*\theta_t^2 - \frac{\gamma^{*2}\theta_t^2}{2},$$

and the novice's payoff is

$$\mathbb{E}\left[\frac{\theta_t}{N}(N^e\gamma^*\theta_t + N^n\xi^*\mu_t) - \frac{\xi^{*2}\mu_t^2}{2}\right] = \frac{N^e}{N}\gamma^*\theta_t^2 + \frac{N^n}{N}\xi^*\theta_t^2 - \frac{\xi^{*2}}{2}\left(\theta_t^2 + \frac{1}{\nu^*}\right).$$

Therefore, the sum of agents' payoffs is

$$N^e\left(\gamma^* - \frac{\gamma^{*2}}{2}\right)\theta_t^2 + N^n\left[\left(\xi^* - \frac{\xi^{*2}}{2}\right)\theta_t^2 - \frac{\xi^{*2}}{2\nu^*}\right].$$

intractable. However, the numerical simulations suggest that, similar to the analysis in Section 4, the continuity of the vector field may lead to the existence of such a sequence.

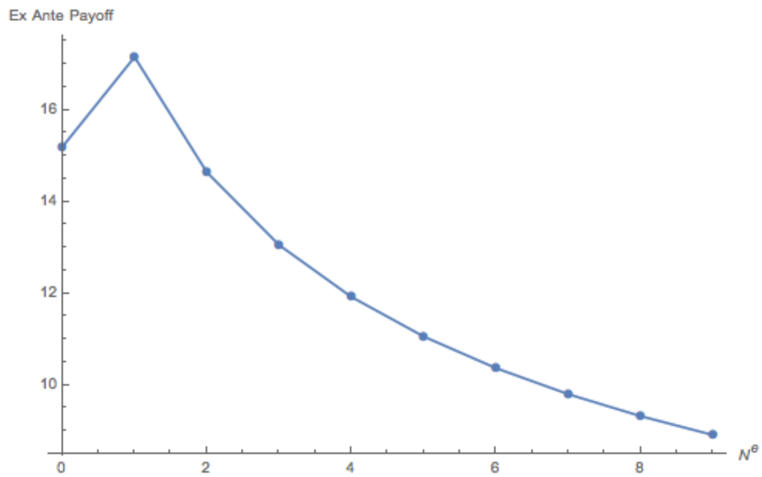


Figure 4: Ex ante total payoff ($N = 10, \delta = 0.98, \theta_t = 3, \kappa_a = \kappa_\theta = \nu_\varepsilon = \nu_\sigma = 1$)

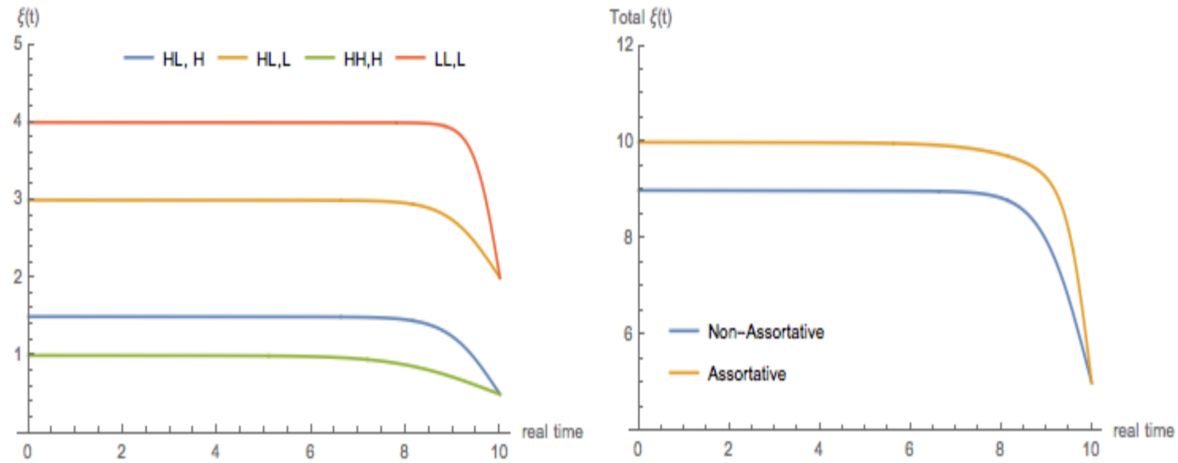
Figure 4 demonstrates the total payoff for a ten-person team as a function of the number of the team members that are experts. The figure shows that replacing a novice with an expert in an all-novice team increases the total welfare. However, each additional expert thereafter decreases the payoff.

The trade-off is clear: Adding additional experts decreases ξ and γ but increases the stationary precision ν^*. In the expression for the total payoff of the agents, the state θ multiplies ξ and γ. Therefore, when θ is high enough, the impact on welfare of the resulting reduction in γ and ξ overrides the impact of the increased informativeness of equilibrium. Therefore, adding an expert decreases the expected payoff.[28]

5.2 Sorting by Productivity

Another interesting design question is how to allocate agents with different effort costs to multiple teams. In our "stage game," the payoffs are additively separable in the types of the agents, and therefore, the sorting of agents into teams is irrelevant to total production. In

[28]One caveat associated with our exercise is that we consider the welfare outcomes when the state θ is fixed at a particular value, instead of evolving according to a random walk. We choose to adopt this approach because, θ being a random walk, the long-run expectation of θ^2 diverges, and therefore, the expression for welfare becomes undefined. Yet, we claim that our exercise is valuable: First, if θ follows a process with *some* mean reversion, the results are likely to be close to this case, and the expectation of θ^2 would not diverge. Second, when initial state is θ_0, for any finite t number of periods, the expectation of θ_t^2 is $\theta_0^2 + t/\nu_\sigma^2$, and therefore, the qualitative discussion remains valid for any initial finite number of periods.



(a) Individual belief sensitivity

(b) Total belief sensitivity

Figure 5: The equilibrium $\xi_i(t)$ under an assortative and a non-assortative team $(N = 2, \tau = 10, (c_H, c_L) = (1, 0.5)$, equal share$)$

contrast, in the dynamic model, this is no longer true. In fact, as discussed above, any agent's belief sensitivity increases if his teammate's cost of effort decreases. The extent to which own belief sensitivity increases in response depends on the agent's own cost. It is intuitive that an agent with a lower cost of effort responds more to a decrease in the effort cost (and therefore to an increase in the belief sensitivity) of his teammate. This intuition indicates that total production (i.e., total belief sensitivity) is maximized under positive assortative matching of workers into teams.

It is easy to illustrate this intuition using the following example with four agents and an equal sharing rule. Consider our main model from Section 3. Suppose that two agents have high effort cost $(c_i = \bar{c})$ while the other two have low effort cost $(c_i = \underline{c})$ $(\bar{c} > \underline{c})$. Suppose that a principal hires all four agents, and he must allocate them into two teams of two agents. What is the allocation that maximizes total output?

For simplicity, we consider the case in which κ_a is large because, as shown in Section 3, in this case ξ_{it} approaches its upper limit, and therefore, the analysis becomes clear. Recall that this upper limit is $\sqrt{\frac{s_i}{c_i}} \sum_{j=1}^{N} \sqrt{\frac{s_j}{c_j}}$. The goal is to maximize total belief sensitivity and, therefore, total output.

First, assume that the output share of each team member is fixed at $1/2$. Then, for an arbitrary team, the total belief sensitivity of effort is given by

$$\xi_{it} + \xi_{jt} \simeq \sqrt{\frac{1}{c_i c_j}} + \frac{1}{2}\left(\frac{1}{c_i} + \frac{1}{c_j}\right). \tag{16}$$

It is immediate from this equation that when shares s_1, s_2, s_3, s_4 are exogenously fixed at $1/2$, the surplus-maximizing sorting is assortative with respect to the effort cost c_i. Figure 5 illustrates this result.

It is also of interest to consider the case in which, once workers are allocated to teams, the shares are optimally chosen to maximize output, as given by Proposition 8 for the limiting case. Under this specification, the total belief sensitivity of effort in the limit as $\kappa \to \infty$ reduces to

$$\xi_{it} + \xi_{jt} \simeq \frac{1}{c_i} + \frac{1}{c_j}.$$

Since this is additively separable in c_i and c_j, the total belief sensitivity is independent of how agents are allocated across teams. In fact, this is not surprising since under these optimal shares, and as $\kappa_a \to \infty$, the effort choices approach their first-best level, which is independent of the types and actions of an agent's teammates. Intuitively, when a low-cost agent on a team is replaced with a high-cost agent, the incumbent teammate is compensated with a higher share such that the decrease in his belief sensitivity due to the increase in the cost of his opponent is exactly offset by an increase in his belief sensitivity due to the increase in his share.

6 Concluding Remarks

The main insights of our paper, along with the tractability of the linear-quadratic-Gaussian framework, present new opportunities for future research. Here, we discuss three potential research directions.

First, the qualitative differences between the results of our paper and those of Bonatti and Hörner (2011) call for a unified analysis of the effect of uncertainty on dynamic team production. Ideally, one could set up a model in which the payoff and informational externalities can be disentangled, and thus, the agents' efforts over time exhibit either strategic substitutability or complementarity conditional on the parameter values.

Second, it may be interesting to consider the case with private feedback. Suppose that the agents receive private feedback in addition to the public feedback. Then, in contrast to our main model, other agents' beliefs become also relevant in making the effort choice, as they convey additional information about the true state. Then, in addition to signal-jamming, there will be an additional signaling incentive for each agent. Extending our model to this case would allow us to analyze further questions such as the optimal design of feedback structure in organizations.

Finally, comparison between the signal-jamming mechanism in our paper and the standard punishment mechanism deserves further exploration. In the infinite-horizon model, there exist many other non-Markovian equilibria that rely on the trigger mechanism to induce cooperation. Following the insight of Sannikov and Skrzypacz (2007), we conjecture that such trigger equilibria would not survive under flexible production, whereas the Markovian equilibrium of our paper would survive and continue to exhibit cooperation. Comparing other aspects of the two mechanisms and analyzing possible combinations of the two are left for future research.

Appendix

Proof of Proposition 2

First, recall that given any pure strategy profile, the Gaussian belief updating process (equations 1 and 2) implies that the agents' beliefs after every period-t history can be summarized by $(\mu_t, (\hat{\mu}_{1t}, \ldots, \hat{\mu}_{Nt}))$, where μ_t and $\hat{\mu}_{it}$ are the mean of the public belief and agent i's private belief in period t, respectively. Furthermore, since every public history is on the equilibrium path, all agents believe that the others have not deviated after any history, and thus, agent i believes that $\hat{\mu}_{jt} = \mu_t$ for all $j \neq i$.

We employ backward induction to prove the proposition. In the last period, each agent maximizes his stage payoff:

$$a_i^*(h_i^T) = \arg\max_a \mathbb{E}_T \left[\Delta s_i \theta \left(a + \sum_{j \neq i} a_{jT}^* \right) \right] - \Delta c_i \frac{a^2}{2} = \arg\max_a s_i \hat{\mu}_{iT} \left(a + \sum_{j \neq i} a_{jT}^* \right) - c_i \frac{a^2}{2}.$$

The first-order condition yields agent i's unique equilibrium effort $a^*_{iT} = (s_i/c_i)\hat{\mu}_{iT} = \xi_{iT}\hat{\mu}_{iT}$.

Now, suppose that the claim of the proposition holds for period $t+1$ and onward—that is, in any equilibrium of the game, agent i plays $a^*_i(h^k_i) = \xi_{ik}\hat{\mu}_{ik}$ for $k = t+1,\ldots,T$, where ξ_{ik} is defined in (4). To solve agent i's optimization problem in period t, we first compute the impact of his effort choice on future public beliefs. Recall (from Section 3.1) that $\bar{a}_{it} = a^*(\bar{h}^t_i)$ is agent i's equilibrium effort at the public history h^t if he had not deviated in the past—that is, the other agents expect agent i to play $\bar{a}_{it}$ at h^t. Suppose that agent i plays a in period t. Then, using (1) and (2), we have

$$\mu_{t+1} - \hat{\mu}_{i,t+1} = \frac{\nu_t}{\nu_{t+1}}(\mu_t - \hat{\mu}_{it}) + \frac{\Delta\kappa_a\kappa_\theta\eta_\varepsilon}{\nu_{t+1}}(a - \bar{a}_{it}) = \frac{\rho_{t+1}}{\rho_t}(\mu_t - \hat{\mu}_{it}) + \rho_{t+1}(a - \bar{a}_{it}). \quad (17)$$

For periods $t+2$ and onward, we use the induction hypothesis that in period $l = t+1,\ldots,T$ agent i plays $a_{il} = \xi_{il}\hat{\mu}_{il}$ while the others expect him to play $\bar{a}_{il} = \xi_{il}\mu_l$. Therefore, we have

$$\mu_k - \hat{\mu}_{ik} = (\mu_{t+1} - \hat{\mu}_{i,t+1})\prod_{l=t+1}^{k-1}\left(\frac{\nu_t}{\nu_{t+1}} - \xi_{il}\rho_{l+1}\right) = (\mu_{t+1} - \hat{\mu}_{i,t+1})\frac{\rho_k}{\rho_{t+1}}\prod_{l=t+1}^{k-1}(1 - \xi_{il}\rho_l), \quad (18)$$

for $k = t+2,\ldots,T$. Substituting (17) into (18) and re-arranging, we obtain

$$\mu_k = \hat{\mu}_{ik} + \left(\frac{1}{\rho_t}(\mu_t - \hat{\mu}_{it}) + (a - \bar{a}_{it})\right)\rho_k\prod_{l=t+1}^{k-1}(1 - \xi_{il}\rho_l). \quad (19)$$

In period t, agent i's optimization problem is given by

$$\begin{aligned} a^*_i(h^t_i) &= \arg\max_a s_i\hat{\mu}_{it}\left(a + \sum_{j\neq i} a^*_j(\bar{h}_{jt})\right) - c_i\frac{a^2}{2} \\ &+ \mathbb{E}_t\left[\sum_{k=t+1}^{T} e^{-r\Delta(k-t)}\left(\left(s_i\xi_{ik} - c_i\frac{\xi^2_{ik}}{2}\right)\hat{\mu}^2_{ik} + s_i\sum_{j\neq i}\xi_{jk}\mu_k\hat{\mu}_{ik}\right) \mid a\right]. \end{aligned}$$

Note that $\hat{\mu}_{ik}$ for $k = t+1,\ldots,T$ is independent of a and has expectation $\hat{\mu}_{it}$. Substituting μ_k with (19), eliminating additive terms that are independent of a, and replacing $\hat{\mu}_{ik}$ with its expectation whenever appropriate, agent i's problem is rewritten as

$$a_i^*(h_i^t) = \max_a \; s_i \hat{\mu}_{it} \left[1 + \sum_{k=t+1}^{T} e^{-r\Delta(k-t)} \sum_{j \neq i} \xi_{jk} \rho_k \prod_{l=t+1}^{k-1} (1 - \xi_{il} \rho_l) \right] a - c_i \frac{a^2}{2}.$$

It is clear that the problem is concave in a and has a unique solution. The first-order condition immediately yields the desired result.

Proof of Proposition 4

First, we show that there exists a unique solution to equation (9), which defines an autonomous first-order nonlinear system of differential equations. Define $\mathbf{u}(t) = (\xi_1(t), \ldots, \xi_N(t), h(t))$; then, $d\mathbf{u}(t) = F(\mathbf{u}(t))$. Given $(\kappa_a, \kappa_\theta, r, \eta_\varepsilon, \eta_\theta)$, F is a Lipschitz continuous function; then, by the Picard-Lindelöf theorem (Theorem 2.2 of Teschl (2012)), there exists a unique solution to this system in the domain $[0, T]$ with the boundary values $\xi_i(\tau) = s_i/c_i$ for all i.

To establish monotonicity, we first observe that $\dot{\xi}_i(\tau) < 0$. Suppose, for a contradiction, that there exist i and $\tilde{t} \in [0, \tau)$ such that $\dot{\xi}_i(\tilde{t}) > 0$. By the continuity of $\dot{\xi}_i(t)$, there exists $\hat{\Delta}_i > 0$ such that $\dot{\xi}_i(t) < 0$ for $t \in (\tau - \hat{\Delta}_i, \tau]$ and $\dot{\xi}_i(\tau - \hat{\Delta}_i) = 0$. Without loss of generality, assume that $i = 1$ attains $\min\{\hat{\Delta}_i | i = 1, \cdots, N\}$, with the convention that $\hat{\Delta}_j = \infty$ if $\dot{\xi}_j(t) < 0$ for all t. This in particular implies that $\dot{\xi}_i(\tau - \hat{\Delta}_1) \leq 0$ for $i \neq 1$.

Next, we claim that $\ddot{\xi}_1(\tau - \hat{\Delta}_1) > 0$. By taking derivatives of both sides of (9) and using $\dot{\xi}_1(\tau - \hat{\Delta}_1) = 0$, we obtain

$$\ddot{\xi}_1(\tau - \hat{\Delta}_1) = \frac{\eta^2 \kappa_\theta^3 \kappa_a}{\left(\nu_0 + \eta t \kappa_\theta^2\right)^2} \left(\frac{s_1}{c_1} \sum_{j=1}^{N} \xi_j(t) - \xi_1(t)^2 \right) - \frac{\eta^2 \kappa_\theta \kappa_a}{\nu_0 + \eta t \kappa_\theta^2} \frac{s_1}{c_1} \sum_{j=1}^{N} \dot{\xi}_j(\tau - \hat{\Delta}_1).$$

Since over $(\tau - \hat{\Delta}_1)$, ξ_1 is strictly decreasing, $\xi_1(\tau - \hat{\Delta}_1) > s_1/c_1$ and, therefore, the first additive term in (9) is positive. Then, $\xi_1(\tau - \hat{\Delta}_1) = 0$ implies by (9) that

$$\frac{s_1}{c_1} \sum_{j=1}^{N} \xi_j(t) - \xi_1(t)^2 > 0$$

This, together with $\dot{\xi}_j(\tau - \hat{\Delta}_1) \leq 0$ establishes our claim that $\ddot{\xi}_1(\tau - \hat{\Delta}_1) > 0$. Now, since $\dot{\xi}_1$ is continuous, $\dot{\xi}_1(\tau-\hat{\Delta}_1)=0$ and $\ddot{\xi}_1(\tau-\hat{\Delta}_1)>0$, there exists $\varepsilon>0$ such that $\dot{\xi}_1(t)>0$ whenever $t\in(\tau-\hat{\Delta}_1-\varepsilon,\tau-\hat{\Delta}_1)$, a contradiction, establishing that for all t, $\dot{\xi}_t\leq 0$.

Monotonicity immediately implies the lower bound on $\xi_i(t)$. Again, $\dot{\xi}_i(t)\leq 0$ implies, by (9), that the term in parentheses on the right-hand side must be positive. That is,

$$\frac{s_i}{c_i}\sum_{j=1}^{N}\xi_j(t)>\xi_i(t)^2.$$

Taking square roots of both sides, summing over i and rearranging establishes the upper bound.

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