Credit Constraint and Excess Return: Case of Chonsei Lease in Korea

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Oct 2017

(Preliminary. Do NOT Cite.)

Abstract

Chonsei lease arrangement, in which up-front deposit is paid at the start and returned at the end of lease without any periodic payment, is quite a unique and also a dominant form of lease in Korea. This paper offers a simple model to explain the existence of chonsei lease arrangement. Chonsei deposit can be thought of as lending from tenant to landlord, and housing service is counted as interest payment. In that perspective, chonsei deposit is cheap as calculated interest rate is higher than market rate. Landlord must have a good investment opportunity to justify chonsei. It is, however, widely perceived that chonsei deposit was mostly used as a leverage of purchasing a house. With credit constraint, this paper suggests that there can be excess return in housing market and that chonsei lease arrangement is facilitated to capture this return. The current demand for housing asset can be restricted by credit constraint and house price can be undervalued. Credit constrained agent may resort to chonsei to fund the money to buy or keep a house. Tenant will ask for high interest payment instead. We note economic environments which facilitate chonsei and explain why this institution is likely to wither in the future.

1 Introduction

Housing asset comprise an important share of households' asset portfolio in Korea. According to Korean Financial Investment Association's report, the share of non financial asset, though in a

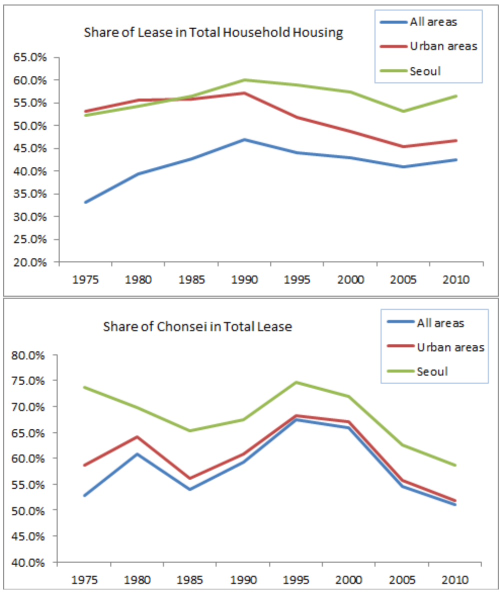

Figure 1: The share of lease and the share of chonsei
Source: Population and Household Census, Korean National Statistical Office

slowly decreasing trend, was over 75% in 2012. This contrasts with around 30% in US and 40% in Japan (both in 2012). Most of non financial assets are real estates, and housing assets are thought to be important chunk in real estate.

In a trade of houses, Korea has a special form of lease arrangement called chonsei.[1] In chonsei lease arrangement, tenants put out up-front deposit to landlords at the start of the lease, which will be returned to tenants at the end of the lease. There will be no monthly payment in a pure form of chonsei.

Chonsei is also a dominant form of lease in Korea. Fig 1 shows the trend of the share of lease in total housing and the importance of chonsei among all lease contracts. According to it, around 45% of households live in houses with lease arrangement. This share is higher in urban areas, and in Seoul, capital city of Korea, it is around 60%. Among these leases, chonsei comprise more than 50% in 1975 and its share is higher in urban areas and in Seoul. Though chonsei's share fluctuated and showed decreasing trend after 1995, it still is higher than 50% in all areas and 60% in Seoul in 2010.[2]

The interesting aspect of chonsei is that it is cheaper than periodic rent scheme. The interest

[1] According to Navaro and Turnbull (2010), this type of lease arrangement, which they call antichresis lease, is also found in nearly all Latin American countries. They also introduce brief history of this lease form.

[2] Recently there emerged mixed form of chonsei and periodic rent, where periodic rent is paid with still significan but less amount of deposit. Here chonsei means its pure form excluding the mixed from.

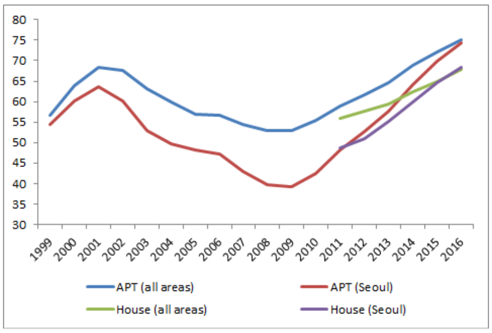

Figure 2: Chonsei deposit to price ratio
Source: Kookmin bank

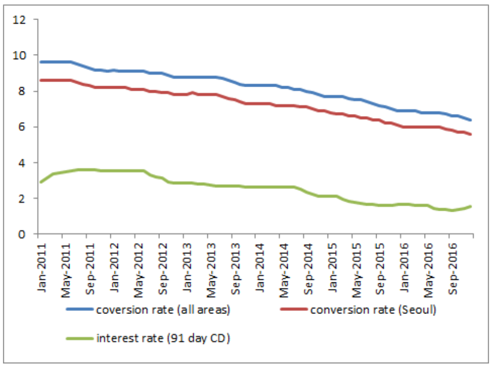

Figure 3: Deposit-rent coversion rate and interest rate
Source: Korea Appraisal Board

payment or investment return from up front deposit should work as rental payment in chonsei contract. Considering market interest rate, however, up-front deposit seems cheap compared with monthly rent. The ratio of up-front deposit to house price was quite low. Fig 1 shows the trend of this ratio. It fluctuates quite a lot and is 70-75% in 2016, but was below 50% in late 2000s. Though we lack official data, it is believed that chonsei deposit to price ratio was much lower in 1970s. Rent to house price ratio may fluctuate due to price increase expectation and it is arguable that certain inflation expectation may justify below 50% chonsei deposit to price ratio.[3] However, if we calculate interest rate comparing chonsei deposit and monthly rent in a similar house, this conversion rate is higher than market interest rate. Fig 1 compares the deposit - rent conversion rate and 91-day CD rate. Even if we consider the spread, the deposit - rent conversion rate seems to be systematically higher than borrowing rate based on housing asset collateral. Though the data covers only very recent period, this seems to be the case when chonsei was more predominant.

This relatively cheap deposit implies that there must be some investment opportunities with

[3] Ambrose and Kim (2003) justifies around 50% level of deposit/price ratio through option pricing model considering defalut risk of landlord. See our literature review.

high return for landlords. That is, landlords want to pay higher interest rate than market rate to grasp those investment opportunities. Literature to explain chonsei contract relies on the investment opportunities with excess return. For example, Kim and Shin (2011) assumes that landlords/entrepreneurs have investment opportunities with potentially higher return, but face imperfect financial market. Chonsei contract may provide conduit of capital for these investment opportunities. However, it is a widely held perception that chonsei deposit was used as a leverage to buy another housing asset especially in 1970s and 80s. This means housing assets themselves provide excess return. This requires explanation of chonsei lease in a housing market itself.

This paper tries to explain the existence of chonsei lease without any pre-imposition of sources of excess return. The key ingredient is credit constraint or imperfect financial market. The basic idea can be sketched as follows. In a period of high income growth, the value of housing is also expected to grow rapidly. With perfect financial market, price increase expectation will increase the current demand for that asset, and asset price will appreciate until the expected return of the asset is equalized to market interest rate. With credit constraint, however, the current demand will be restricted because the purchase cannot be funded. This makes housing asset undervalued, and excess return of housing asset is possible. This excess return of housing asset can be shared between landlord and tenant through chonsei contract. A landlord can keep a house (or buy another house) by borrowing money from tenant. Tenant instead ask for higher interest payment which is in a from of housing service. That makes the chonsei deposit cheaper compared with monthly rent given market interest rate.

To succinctly illustrate this idea, this paper provides a simple two period consumption choice model with no uncertainty when a fixed amount of housing asset is exogenously given. We assume an extreme form of financial market imperfection for simplicity: no borrowing is possible, which will be relaxed in our extensions. If financial market is perfect, price is adjusted so that everyone is indifferent between selling (buying), leasing (renting) through chonsei, and leasing (renting) through periodic rent. If financial market is imperfect, though, the first period house price can be undervalued. Some house owners with low first period income wants to sell houses for consumption smoothing purpose even at undervalued price. This creates room for chonsei lease arrangement. If chonsei lease is available, house owners lease houses through chonsei to achieve partial consumption smoothing and keep the excess return from housing asset. Tenants will ask for higher interest rate to share some of the excess return of housing assets. Thus chonsei deposit is cheaper considering market interest rate and periodic rent. Since the same function cannot be achieved through period rent contract, chonsei lease can be a predominant form of lease contract.

We also considered an alternative form of credit constraint and analyzed the effect of the extent of credit constraint. Suppose a certain portion of house price can be borrowed with housing asset being used as collateral. The relaxation of credit constraint as higher portion of house can be borrowed will increase both the current house price and chonsei deposit. Eventually, the room for chonsei lease contract may disappear. As is shown in Fig 1, chonsei lease is in a declining trend after 1995 though it still comprises majority of lease arrangement.[4] When chonsei deposit increases, many landlords recently wants to charge monthly rent for increased portion of deposit. Mixed form of chonsei and periodic rent becomes more prevalent. Development of financial market, especially mortgage lending may play an important role in that.

■[The paper's model also has empirical implication to be tested. There were some empirical trials to relate chonsei deposit/price ratio to price increase expectation. This paper implies that price increase expectation also has correlation with the prevalence of lease arrangement, not just with deposit/price ratio. With imperfect financial market, current undervaluation of housing asset or high price increase expectation will lead to more prevalence of chonsei lease arrangement instead of monthly rent scheme.]

The paper is organized as follows. Section ? review the related literature. Section ? introduces the model and analysis of it follows in Section ?. The implication of the model is briefly discussed in Section ?. Section ? pursues several extensions. Then conclusion follows.

1.1 Related Literature

Most of Korean literature on chonsei focuses on the level of chonsei deposit taking the existence of chonsei market as given. They tend to empirically test the relationship between price increase expectation and chonsei deposit to house price ratio. For example, Lee (2013) empirically test the theoretically predicted relationship between the two using panel data. Similar literature focusing on the level of chonsei deposit to house price ratio and other variables includes Kim et.al (1998), Cho (2005), and Son et.al (2011). The ratio of chonsei deposit to house price was investigated in

[4] Once chonsei becomes a dominant form of lease, it may not easily disappear even when economic environment has changed. To change the lease arrangement from chonsei to montly rent, chonsei deposit should be returned. Many landlords may have liquidity constraint to do that.

a different angle in Ambrose and Kim (2003). Based on option pricing model, equilibrium chonsei deposit is dependent on landlord's default risk, which is the probability house price goes down below chonsei deposit at the end of lease. This paper, on the contrary, is more interested in why chonsei lease arrangement emerges and what economic environment facilitates its emergence.

To my knowledge, there are only three papers focusing on the emergence of chonsei lease arrangement. As mentioned before, Kim and Shin (2011) considers chonsei lease arrangement as a conduit of capital to profitable investment projects which landlords/entrepreneurs have. They emphasize efficiency enhancing role of chonsei lease. The difference with this paper is already mentioned in the introduction. Navaro and Turnbull (2010) considers antichresis leases in Bolivia, which are exactly the same arrangement as chonsei, and explains its emergence as an incentive scheme to maintain the quality of the property. Depending on the relative importance of efforts between from landlord and from tenant for the maintenance of the property, it is determined whether periodic rent or antichresis is optimal scheme. They focuses on the variation of contract depending on the type of property. This paper disregards the type of property, but focuses on credit constraint as a driving force of chonsei lease. Similar reason was investigated by Kim (2013). He considers contractual incentive of landlords and tenants when they choose between chonsei lease and periodic rent. Given other economic environment including house price, he searches for reasons to choose chonsei lease over periodic rent scheme. Though he listed excess return of house and credit constraint for house purchase as reasons to choose chonsei lease, excess return of the house is exogenously imposed in that model. In contrast to the decision model of Kim, this paper considers an equilibrium model. We endogenously derive excess return of housing asset, which provides a necessary condition of the emergence of chonsei lease.

2 Model

We consider a simple intertemporal consumption decision model with two periods, $t = 0, 1$ without any uncertainty.

Agents There are continuum of agents of unit mass with income w_0^i and w_1 in each period respectively, $i \in [0, 1]$. The first period income w_0^i is distributed with distribution function $F(w)$ over $[0, \overline{w}]$, while the second period income is the same for every agent. Given an income profile, an agent i tries to allocate her consumption over two periods to maximize utility function

$$u\left(c_0^i\right) + \beta u\left(c_1^i\right),$$

with $u' > 0$ and $u'' < 0$.

Imperfect Financial Market An agent i can reallocate her income in period 0 to period 1 through saving with interest rate r. However, financial market is imperfect and borrowing is not possible. This describes a situation where consumer lending market was not fully developed as in Korea until 1990s. Thus, there is no way to reallocate her income from period 1 to period 0, and agents with low income in period 0 cannot smooth her consumption. We will consider alternative and less extreme assumption for imperfect financial market in our extensions.

Housing Asset There exist identical housing assets with mass s with $s < 1$. Agents are randomly endowed with housing assets independent of their income levels, and we assume each agent can possess up to one unit of housing asset. That is, s portion of agents have one unit of housing asset each while $1 - s$ portion do not. Each unit of housing asset has consumption value H, and we assume its period 1 price is fixed at p_1 including its consumption value.

Housing assets can be traded at the start of period 0. House owners are willing to sell their houses for consumption smoothing purpose if her period 0 income is low. Non house owners are willing to purchase a house as an alternative way of saving.

Housing Lease Contract We consider two possible housing lease arrangement. One is periodic rental payment scheme where the tenant pays a non-returnable rent for their use of house. The other is chonsei contract, where a tenant pays up-front deposit which will be returned at the end of contract. In a nutshell, interest income generated from up-front deposit will work as periodic rent in the former scheme.

3 Analysis

We basically analyze housing market in period 0 and tries to obtain an equilibrium price. We can think of three types of markets related to housing asset. One is a market for purchase and the other two are lease markets with each type of lease arrangement. We will obtain equilibrium purchase price p_0, chonsei deposit p_r, and rental price R.

3.1 Benchmark: Perfect Financial market where borrowing is also possible with interest rate r

As a benchmark, we first analyze the market when borrowing is also possible with the same interest rate r. With perfect financial market, an agent i will allocate her total wealth to consumption in each period to optimally smooth her consumption path. Agent i's optimization problem is

$$\max_{c_0^i, c_1^i} u\left(c_0^i\right) + \beta u\left(c_1^i\right)$$
$$s.t.\ \ c_0^i + \frac{c_1^i}{1+r} = W^i$$

where W^i is the present value of wealth. Optimal consumption is obtained by

$$u'\left(c_0^i\right) = \beta\left(1+r\right) u'\left(c_1^i\right) \tag{1}$$

and the constraint. The present value of wealth W^i is dependent on the ownership of housing asset. Non house owner's present value of wealth, W_N^i is

$$W_N^i = w_0^i + \frac{w_1}{1+r}, \tag{2}$$

and house owner's present value of wealth, W_O^i, contains the consumption value of housing in period 0 and its price in period 1 additionally,

$$W_O^i = w_0^i + H + \frac{w_1 + p_1}{1+r}. \tag{3}$$

Optimal consumption path is dependent on w_0^i which varies by agent, but its type only depends on the ownership of housing asset. From now on, we omit superscript i if it does not cause confusion. Let $\left(c_0^O, c_1^O\right)$ and $\left(c_0^N, c_1^N\right)$ be optimal consumption path for house owners and non house owners respectively.

Lemma 1 *Optimal consumption path* $\left(c_0^O, c_1^O\right)$ *and* $\left(c_0^N, c_1^N\right)$ *are determined by (1) and budget* *constraint with wealth defined by (3), and (2) respectively.*

Since financial market is perfect, purchase or sale of a housing asset should not affect W. If not, house price will be adjusted. For example, If price is high enough so that sale of a house increases one's W, every house owner is willing to sell their houses and price will fall. Therefore, equilibrium house price p_0^* is

$$p_0^* = H + \frac{p_1}{1+r}. \tag{4}$$

By the same logic, rent R is paid for consumption value of housing and interest income from chonsei deposit p_r should cover periodic rent,

$$\frac{r}{1+r} p_r^* = R^* = H. \tag{5}$$

Proposition 1 *With perfect financial market, equilibrium house price, rent, and chonsei deposit are determined by (4) and (5).*

Note that there is no separate role in chonsei lease arrangement. Even without chonsei arrangement, periodic rental contract will be enough to cover all lease arrangement. As we can see later, this is no longer true once we introduce imperfectness of financial market.

If we assume π as rate of house price increase, that is $p_1 = (1+\pi)\, p_0$, chonsei deposit and house price should satisfy

$$\frac{p_r^*}{p_0^*} = \frac{r-\pi}{r}.$$

Corollary 1 *With perfect financial market, A ratio of chonsei deposit to house price decreases in the expected house price inflation and increases in interest rate.*[5] *When there is no price change, chonsei deposit and house price would be the same.*

This is the implication which was quite often tested in the literature. Our interest, in contrast to those literature, is to explain the emergence and role of chonsei lease arrangement.

3.2 Imperfect Financial Market: No Borrowing Available

If there is no borrowing available, house owners with low w_0 may be willing to sell a house with price lower than p_0^*. This will lower her present value of wealth, but can be beneficial for the

[5] Note that π cannot exceed r. If it does, no house owner would sell the house and π will decrease.

consumption smoothing purpose. We will first discuss the possibility of an equilibrium where house price is lower than p_0^* without considering chonsei lease market. Then we discuss how chonsei lease market emerges and how it affects house purchase market.

3.2.1 Housing Market without Chonsei Lease Arrangement

If periodic rent is the only possible arrangement in the lease market, rent will still be $R^* = H$. Whether borrowing is possible or not, consumption value of housing H is only traded in this lease market.

In a market for purchase, price of house asset should be lower than p_0^*. As explained above, some house owners are willing to sell house for consumption smoothing purpose even if a sale will decrease the present value of wealth. From buyer's side, no one is willing to buy a house if a purchase decreases the present value of wealth. Buyers can always save the money to keep the same wealth level and achieve consumption smoothing. Thus, if we denote equilibrium price as p_0^I, then

$$p_0^I \leq p_0^*.$$

We will investigate house owners' sales decision and non house owner's purchase decision in turn, and then market equilibrium. Let us consider house owners' sales decisions first. If one keeps a house, the present value of wealth is W_O in (3). If optimal consumption smoothing is achievable, one will choose the optimal consumption path $\left(c_0^O, c_1^O\right)$. Optimal consumption smoothing is achievable without borrowing if period 0 available income, $w_0 + H$, is greater than the optimal consumption c_0^O. It is also possible that c_0^O is so little that an agent has to consume H in period 0. We exclude this possibility by assuming w_1 is large enough. If period 0 income is lower than the optimal consumption, however, there is no way to consume future income in advance. Then it is optimal to consume all of one's income in each period.

Lemma 2 *If a house owner keeps a house, her maximized utility $U_O^0(w_0)$ is*

$$U_O^0(w_0) = \begin{cases} u\left(c_0^O\right) + \beta u\left(c_1^O\right) & \text{if } w_0 + H \geq c_0^O \\ u(w_0 + H) + \beta u\left(w_1 + p_1\right) & \text{if } w_0 + H < c_0^O \end{cases}. \tag{6}$$

If a house owner sells a house, the present value of wealth W_O^S is

$$W_O^S = w_0 + p_0 + \frac{w_1}{1+r}. \tag{7}$$

Optimal consumption path $\left(c_0^{OS}, c_1^{OS}\right)$ is determined by (1) and budget constraint with wealth of (7). If period 0 income including sales price of a house, $w_0 + p_0$, is greater than c_0^{OS}, optimal consumption path can be achieved. Otherwise, all available income will be consumed in each period.

Lemma 3 *If a house owner sells a house, her maximized utility $U_O^S(w_0)$ is*

$$U_O^S(w_0, p_0) = \begin{cases} u\left(c_0^{OS}\right) + \beta u\left(c_1^{OS}\right) & \text{if } w_0 + p_0 \geq c_0^{OS} \\ u(w_0 + p_0) + \beta u(w_1) & \text{if } w_0 + p_0 < c_0^{OS} \end{cases}. \tag{8}$$

A house owner will make a sales decision by comparing $U_O^0(w_0)$ and $U_O^S(w_0, p_0)$. Note that the present value of wealth is greater when a house is kept as $p_0 \leq p_0^*$,

$$W_O \geq W_O^S.$$

The advantage of selling a house is more consumption in period 0. Consider a period 0 income level w_0' such that $w_0' + H = c_0^{OS}$. If $w_0 > w_0'$, consumption in period 0 with sale of a house is still lower than without sale. Thus a house owner is better off without sale, $U_O^0(w_0) > U_O^S(w_0, p_0)$. Moreover, $\frac{d}{dw_0}U_O^0 > \frac{d}{dw_0}U_O^S$ if $w_0 < w_0'$. Since the period 0 consumption without sale is lower that with sale, marginal utility of period 0 consumption is greater.

As Fig 4 shows, there exists $\widehat{w_O}(p_0)$ such that house owner is willing to sell their houses if $w_0 < \widehat{w_O}(p_0)$.[6] This threshold income level will determine the quantity supplied in the house market. Specifically, supply is given by $sF\left(\widehat{w_O}(p_0)\right)$. Note that the graph U_O^S moves up as p_0 increases. Supply $sF\left(\widehat{w_O}(p_0)\right)$ is increasing in p_0.

Proposition 2 *There exists threshold period 0 income level $\widehat{w_O}(p_0)$ such that house owners are willing to sell their houses if $w_0 \leq \widehat{w_O}(p_0)$. Supply in a house market is given by $sF\left(\widehat{w_O}(p_0)\right)$, which is increasing in p_0.*

Similarly, we can investigate the purchase decision of non house owners. If one does not buy a house, her present value of wealth is W_N in (2). If her period 0 income exceeds period 0 optimal

[6] Such $\widehat{w_O}$ exists as long as p_0 is not too low, i.e. $u(p_0) + \beta u(w_1) > u(H) + \beta u(w_1 + p_1)$.

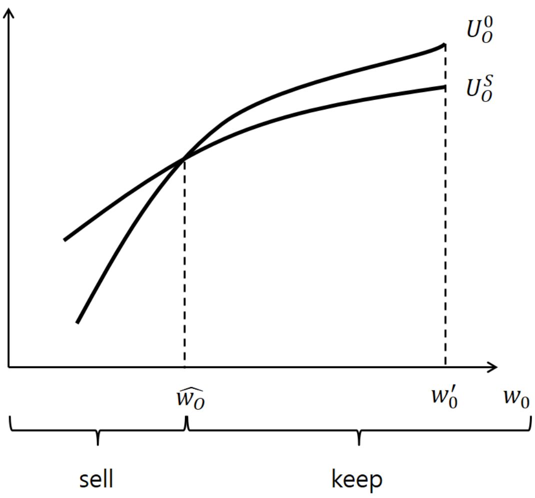

Figure 4: Comparison of U_O^0 and U_O^S

consumption, optimal consumption path will be chosen. Otherwise, all available income will be consumed in each period. If one buys a house, her present value of wealth W_N^B is

$$W_N^B = w_0 - p_0 + H + \frac{w_1 + p_1}{1+r}, \tag{9}$$

and optimal consumption path $\left(c_0^{NB}, c_1^{NB}\right)$ is determined by (1) and budget constraint with wealth of (9). If her period 0 income after paying the house price and getting consumption value of house instead, $w_0 - p_0 + H$, exceeds period 0 optimal consumption, optimal consumption path will be chosen. Otherwise, all available income will be consumed in each period. Let U_N^0 and U_N^B be her maximized utility with and without purchase of a house respectively. Then they are expressed as in the following Lemma.

Lemma 4 *Non house owners' utility without purchase of a house $U_N^0(w_0)$ and with purchase of a house $U_N^B(w_0, p_0)$ are as follows.*

$$\begin{aligned}
U_N^0(w_0) &= \begin{cases} u\left(c_0^N\right) + \beta u\left(c_1^N\right) & \text{if } w_0 \geq c_0^N \\ u(w_0) + \beta u\left(w_1\right) & \text{if } w_0 < c_0^N \end{cases} \\
U_N^B(w_0, p_0) &= \begin{cases} u\left(c_0^{NB}\right) + \beta u\left(c_1^{NB}\right) & \text{if } w_0 - p_0 + H \geq c_0^{NB} \\ u(w_0 - p_0 + H) + \beta u\left(w_1 + p_1\right) & \text{if } w_0 - p_0 + H < c_0^{NB} \end{cases}
\end{aligned}$$

Non house owners make a purchase decision by comparing U_N^0 and U_N^B. Note that buying a

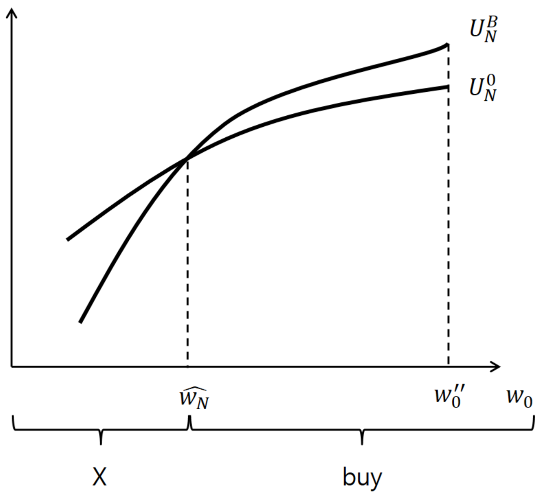

Figure 5: comparison of U_N^0 and U_N^B

house increases one's wealth,

$$W_N^B \geq W_N$$

, but has an disadvantage of reducing one's period 0 available income. Consider a period 0 income level w_0'' such that $w_0'' - p_0 + H = c_0^N$. If $w_0 > w_0''$, consumption in period 0 with purchase of a house is higher than without purchase. Thus a non house owner is better off with purchase, $U_N^B(w_0, p_0) > U_N^0(w_0)$. Moreover, $\frac{d}{dw_0}U_N^B > \frac{d}{dw_0}U_N^0$ if $w_0 < w_0''$. Since the period 0 consumption with purchase is lower if $w_0 - p_0 + H < c_0^N$, marginal utility from period 0 consumption is greater.

As Fig 5 shows, there exists $\widehat{w_N}(p_0)$ such that non house owner is willing to buy a house if $w_0 \geq \widehat{w_N}(p_0)$. Thus the demand in a house market is $(1-s)\{1 - F(\widehat{w_N}(p_0))\}$. Since the graph of U_N^B moves down as p_0 increase, this demand is decreasing in p_0.

Proposition 3 *There exists threshold period* 0 *income level* $\widehat{w_N}(p_0)$ *such that non house owners are willing to buy a house if* $w_0 \geq \widehat{w_N}(p_0)$. *The demand for houses is given by* $(1-s)\{1 - F(\widehat{w_N}(p_0))\}$, *which is decreasing in* p_0.

Combining house owners' and non house owners' decisions, we can define an equilibrium in a house market. Market equilibrium is a house price which equalizes supply and demand.

Definition 1 *Equilibrium price of a house with imperfect financial market* p_0^I *is defined as one satisfying*

$$sF\left(\widehat{w_O}\left(p_0^I\right)\right) = (1-s)\left\{1 - F\left(\widehat{w_N}\left(p_0^I\right)\right)\right\}.$$

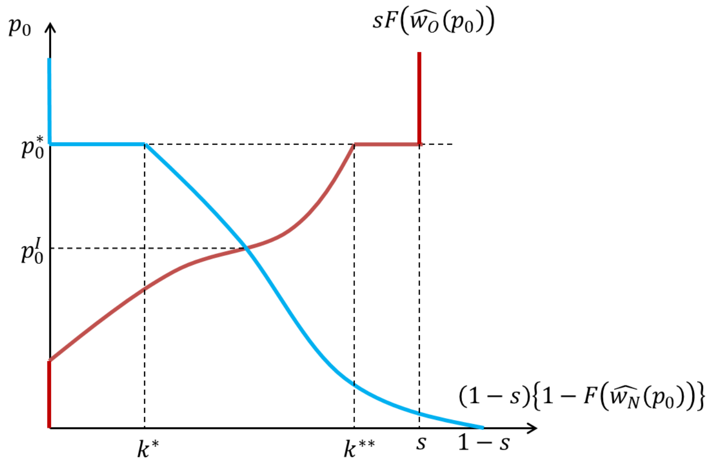

Figure 6: Equilibrium of House Market

Fig 6 illustrates an equilibrium price in this market. As the figure suggests, there exists an equilibrium price and it is unique as supply (demand) is monotonically increasing (decreasing) in p_0. The equilibrium price should be strictly lower than that with perfect financial market as long as k^{**} is bigger than k^*. Some non house owners, if their period 0 income is high enough, are indifferent between buying and not buying a house even if wealth level stays the same. This mass is denoted by k^* in Fig 6.[7] Some house owners, if their period 0 income is high enough, never sell houses if it decreases wealth level. The remaining mass are willing to sell houses even if p_0 is lower than but very close to p_0^*. This mass is k^{**} in the figure.[8] If k^* is smaller than k^{**}, demand falls short of supply at p_0^*. Thus the equilibrium price is lower than p_0^*.

Proposition 4 *Equilibrium price p_0^I exists and it is unique. It is lower than equilibrium price with perfect financial market, $p_0^I \leq p_0^*$.*

The difference between p_0^I and p_0^* can be thought of as 'illiquidity penalty' of a house asset. An asset, which can provide liquidity at a necessary point of time, enjoys liquidity premium in its price and thus is overvalued. (Holmstrom and Tirole 2001) A house asset here is illiquid and faces an opposite effect. Credit constraint make low income house owners sell their houses while restricting the demand for it. Thus house in period 0 is undervalued considering its consumption value H and future price prospect p_1. This undervaluation of a house asset creates excess return. By excess

[7]Specifically, $k^* = (1-s)\{1-F(w_0^*)\}$ where $w_0^* - p_0^* + H = c_0^N$.
[8]$k^{**} = sF(w_0^{**})$ where $w_0^{**} + H = c_0^O$.

return, we mean higher return than market interest rate r. One with enough credit (high income in period 0) can enjoy higher present value of wealth by keeping or buying a house.

We can apply several comparative statics, which are summarized in the following proposition.

Proposition 5 *Equilibrium price p_0^I is affected by the parameters of model as follows.*

i) If s increases, p_0^I decreases with the same p_0^*.

ii) If p_1 increases, both p_0^* and p_0^I increase.

iii) If r increases, p_0^* decreases and p_0^I weakly decreases.

iv) If w_1 increases, p_0^I decreases with the same p_0^*.

v) If income follows a distribution function G which is first order stochastically dominated by F, p_0^I decreases.

Proof. See the appendix. ■

If an asset is more abundant, its price goes down in i). Higher price in the future increases the current price in ii). If r is higher, opportunity cost of buying house increases while that of selling house decreases, which will reduce the price. Both iv) and v) show the same qualitative result. If the current income is lower in v) or future income is higher in iv), there will be more demand for liquidity in period 0 for consumption smoothing purpose. Thus illiquidity penalty will increase or housing asset is more undervalued.

The excess return of an housing asset also creates a room for another lease arrangement, chonsei. Chonsei arrangement can be thought of as lending agreement from tenant to landlord. Interest payment is made in a form of housing consumption.[9] Through this contract, the excess return of housing asset can be shared. Landlord can keep the house and enjoy the excess return while obtaining a certain level of consumption smoothing by borrowing from tenant though the achieved consumption smoothing may not be complete. Tenant, who provide this valuable credit, will ask for higher interest payment instead and get a certain share of excess return. Thus chonsei deposit will be cheap considering market interest rate and housing consumption value. This arrangement may drive out periodic rent arrangement, which does not provide an opportunity to share this excess return. Moreover, chonsei deposit can relax credit constraint when purchasing a house. Some non

[9] This character of chonsei contract, or house repo contract, is pointed out by Kim and Shin(2011). The difference is that they presuppose excess return from other sectors while this paper creates excess return in the housing sector itself. See literature review for discussion.

house owners can buy a house but rent it through chonsei arrangement.[10] This will relax their credit constraint which can make the purchase beneficial. We will investigate this possibility in our next discussion.

3.2.2 Possibility of Chonsei Lease Arrangement

We still keep periodic rental market and its rent is $R^* = H$ as with perfect financial market. As argued above, periodic rent is just an exchange between housing consumption value H and rent R. This does not change whether borrowing is possible or not.

We consider a necessary condition of chonsei deposit p_r for chonsei lease market to exist.[11] First, chonsei deposit should be low to attract tenants,

$$p_r < p_r^* = \frac{1+r}{r}H. \tag{10}$$

Otherwise, tenants would prefer periodic rent. Chonsei requires up front deposit, which is a big disadvantage when borrowing is not available. To make up for this disadvantage, deposit should be cheap so that tenants' present value of wealth can increase.

Second, however, chonsei deposit cannot be too low. If it is too low such that $p_0 \geq \frac{r}{1+r}p_r + \frac{1}{1+r}p_1$, then house owners would rather sell their houses than lease them. Selling houses would increase the present value of wealth and also be more helpful for consumption smoothing as $p_0 > p_r$. Thus,

$$p_0 < \frac{r}{1+r}p_r + \frac{1}{1+r}p_1. \tag{11}$$

By combining (10) and (11), we have a necessary condition for the existence of chonsei market.

Lemma 5 *If chonsei market is to exist, it is necessary that chonsei deposit p_r satisfies*

$$\frac{1+r}{r}p_0 - \frac{1}{r}p_1 < p_r < \frac{1+r}{r}H.$$

Note that $\frac{1+r}{r}p_0^I - \frac{1}{r}p_1 < \frac{1+r}{r}H$ as $p_0^I < p_0^*$. Thus the room for chonsei lease market is created

[10] Note that we assume an agent can have at most one house. If they can own multiple houses, some will get chonsei deposit and use this for the purchase of another house.

[11] Though chonsei lease arrangement can exist with perfect finanical market, it does not provide different roles from periodic rent arragement. By the existence of chonsei lease market, we mean chonsei provides a meaningfully different role.

by the undervaluation of house in period 0 due to imperfect financial market. We now consider house owners' and non house owners incentives to participate in this chonsei lease market.

We first consider a house owner's choice. If chonsei lease is available, a house owner has one more option, leasing a house through chonsei. If she keeps or sells her house, her maximized utility is as described in (**??**) and (**??**). If she leases a house through chonsei, her present value of wealth is

$$W_O^L = w_0 + \frac{r}{1+r} p_r + \frac{w_1 + p_1}{1+r}. \tag{12}$$

If financial market is perfect and wealth can be freely allocated, her optimal consumption path $\left(c_0^{OL}, c_1^{OL}\right)$ is determined by the condition (1) and budget constraint with wealth level (12). Her available income in period 0 is $w_0 + p_r$. If it exceeds c_0^{OL}, optimal consumption path $\left(c_0^{OL}, c_1^{OL}\right)$ will be followed. Otherwise, available income is consumed in each period.

Lemma 6 *If a house owner leases a house through chonsei contract, her utility* U_O^L *is*

$$U_O^L(w_0, p_r) = \begin{cases} u\left(c_0^{OL}\right) + \beta u\left(c_1^{OL}\right) & \text{if } w_0 + p_r \geq c_0^{OL} \\ u(w_0 + p_r) + \beta u\left(w_1 + p_1 - p_r\right) & \text{if } w_0 + p_r < c_0^{OL} \end{cases}.$$

If the necessary condition in Lemma 5 holds, wealth level is higher in the order of keeping, leasing, and selling a house,

$$W_O \geq W_O^L \geq W_O^S.$$

The advantage of selling or leasing a house is to secure more available income in period 0.

Consider period 0 income level w_0''' such that $w_0''' + H = c_0^{OL}$. If $w_0 \geq w_0'''$, keeping a house is a better choice than leasing. Moreover, $\frac{d}{dw_0}U_O^0(w_0) > \frac{d}{dw_0}U_O^L(w_0, p_r)$ if $w_0 < w_0'''$, as utility increase of revamping period 0 consumption is larger when period 0 consumption is lower. Then there exists a threshold income level $\widehat{w_O^1}(p_r)$ such that $U_O^0(\widehat{w_O^1}(p_r)) = U_O^L\left(\widehat{w_O^1}(p_r), p_r\right)$. Keeping a house is better than leasing if $w_0 > \widehat{w_O^1}(p_r)$.

By the same token, there also exists another threshold income level $\widehat{w_O^2}(p_0, p_r)$ such that $U_O^L\left(\widehat{w_O^2}(p_0, p_r), p_r\right) = U_O^S\left(\widehat{w_O^2}(p_0, p_r), p_0\right)$. Leasing a house is better than selling if $w_0 > \widehat{w_O^2}(p_0, p_r)$.

If there is to be any supply in chonsei lease market, $\widehat{w_O^1}(p_r)$ should be greater than $\widehat{w_O^2}(p_0, p_r)$ so that house owners in the income interval $\left[\widehat{w_O^2}(p_0, p_r), \widehat{w_O^1}(p_r)\right]$ choose to lease the house.

This is illustrated in Fig 7. Recall equilibrium house price p_0^I without chonsei market. Given

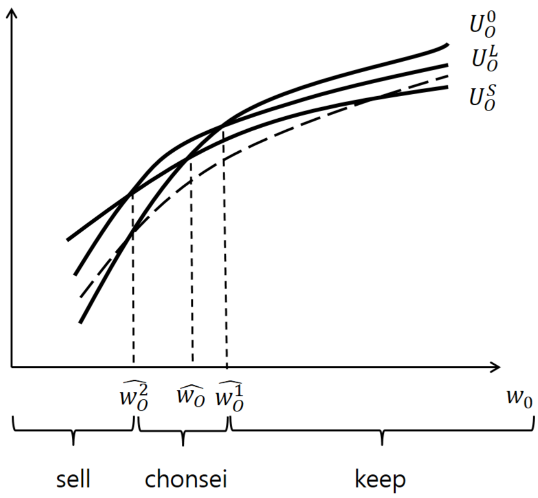

Figure 7: House owners' decisions

p_0^I, U_O^S has a unique intersection with U_O^0 at $\widehat{w_O}\left(p_0^I\right)$. The graph of U_O^L moves up as p_r increases. If p_r is too low, U_O^L is below U_O^S and U_O^0 at their intersection and no one will lease house through chonsei. Thus p_r should be higher than a certain level, if there is to be supply in chonsei market,

Lemma 7 *Some house owners are willing to lease their houses If* $p_r > \underline{p_r}\left(p_0^I\right)$ *where*

$$U_O^L\left(\underline{p_r}\left(p_0^I\right)\right) = U_O^0\left(\widehat{w_O}\left(p_0^I\right)\right) = U_O^S\left(\widehat{w_O}\left(p_0^I\right), p_0^I\right).$$

As Fig 7 illustrates, period 0 income level is divided into three intervals if $p_r > \underline{p_r}\left(p_0^I\right)$. House owners sell their houses if w_0 is low, lease them if w_0 is in the middle range, and keep them if w_0 is high enough.

Proposition 6 *If* $p_r > \underline{p_r}\left(p_0^I\right)$*, a house owner sells a house if* $w_0 < \widehat{w_O^2}(p_0, p_r)$*, leases it if* $\widehat{w_O^2}(p_0, p_r) \leq w_0 < \widehat{w_O^1}(p_r)$*, and keeps it if* $w_0 \geq \widehat{w_O^1}(p_r)$*.*

Thus the supply of the house market and chonsei market are $sF\left(\widehat{w_O^2}(p_0, p_r)\right)$ and $s\left[F\left(\widehat{w_O^1}(p_r)\right) - F\left(\widehat{w_O^2}(\right.\right.$ respectively. As p_0 increases, the graph of U_O^S moves up. This lead to increase of supply in the house market, but decrease of supply in the chonsei market. As p_r increases, the graph of U_O^L moves up. This lead to increase of supply in the chonsei market, but decrease of supply in the house market. Thus the law of supply holds in each market.

Similar argument can be applied to non house owners' choices. One more available option is to rent a house through chonsei. Then her present value of wealth is

$$W_N^L = w_0 - \frac{r}{1+r} p_r + H + \frac{w_1}{1+r}. \tag{13}$$

Let $\left(c_0^{NL}, c_1^{NL}\right)$ be optimal consumption path determined by (1) and budget constraint with wealth level (13). Non house owner's utility when renting a house through chonsei is similarly defined by comparing her available income $w_0 - p_r + H$ and optimal consumption c_0^{NL} in period 0.

Lemma 8 *If a non house owner rents a house through chonsei contract, her utility U_N^L is*

$$U_N^L(w_0, p_r) = \begin{cases} u\left(c_0^{NL}\right) + \beta u\left(c_1^{NL}\right) & \text{if } w_0 - p_r + H \geq c_0^{NL} \\ u(w_0 - p_r + H) + \beta u(w_1 + p_r) & \text{if } w_0 - p_r + H < c_0^{NL} \end{cases}.$$

If the necessary condition in Lemma 5 holds, wealth level is higher in the order of buying, renting, and not buying,

$$W_N^B \geq W_N^L \geq W_N^0.$$

Advantage of not buying or renting is that one can secure more available income in the first period. Consider income level w_0'''' such that $w_0'''' - p_0 + H = c_0^{NL}$. If $w_0 \geq w_0''''$, period 0 consumption is higher even with purchase of a house and buying is better than renting. Moreover, $\frac{d}{dw_0} U_N^B(w_0, p_0) > \frac{d}{dw_0} U_N^L(w_0, p_r)$ if $w_0 < w_0''''$. There exists a threshold income level $\widehat{w_N^1}(p_0, p_r)$ such that buying is preferred to renting if income is higher than that level. Similarly, there also exists a threshold income $\widehat{w_N^2}(p_r)$ such that renting is preferred to not buying.

If there is to be any demand for chonsei, $\widehat{w_N^1}(p_0, p_r)$ should be greater than $\widehat{w_N^2}(p_r)$ so that house owners in the income interval $\left[\widehat{w_N^2}(p_r), \widehat{w_N^1}(p_0, p_r)\right]$ choose to rent the house.

Fig 8 illustrates this. Given p_0^I, U_N^B has a unique intersection with U_N^0 at $\widehat{w_N}\left(p_0^I\right)$. The graph of U_N^L moves down as p_r increases. If p_r is too high, U_N^L is below U_N^B and U_N^0 at their intersection and no one will lease house through chonsei. Thus p_r should be lower than a certain level, if there is to be demand in chonsei market,

Lemma 9 *Some non owners are willing to rent houses if $p_r < \overline{p_r}\left(p_0^I\right)$ where*

$$U_N^L\left(\widehat{w_N}\left(p_0^I\right), \overline{p_r}\left(p_0^I\right)\right) = U_N^0\left(\widehat{w_N}\left(p_0^I\right)\right) = U_N^B\left(\widehat{w_N}\left(p_0^I\right), p_0^I\right).$$

As Fig 8 illustrates, period 0 income level is divided into three intervals if $p_r < \overline{p_r}\left(p_0^I\right)$. Non

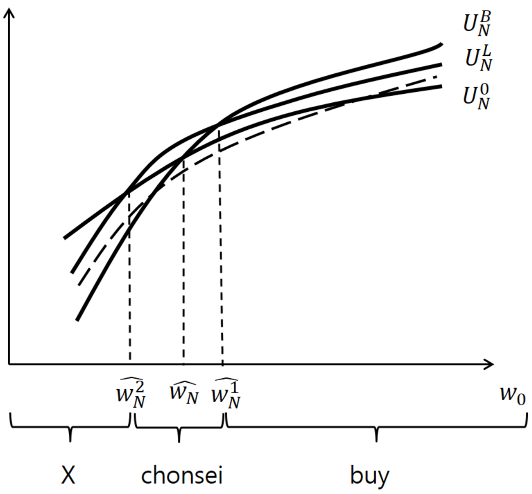

Figure 8: Non house owners' decisions

house owners buy their houses if w_0 is high enough, rent them if w_0 is in the middle range, and become inactive if w_0 is low.

Proposition 7 *If $p_r < \overline{p_r}\left(p_0^I\right)$, a non house owner becomes inactive if $w_0 < \widehat{w_N^2}\left(p_r\right)$, rents a house if $\widehat{w_N^2}\left(p_r\right) \leq w_0 < \widehat{w_N^1}\left(p_0, p_r\right)$, and buys one if $w_0 \geq \widehat{w_N^1}\left(p_0, p_r\right)$.*

Thus the demand of the house market and the chonsei market are $(1-s)\left[1 - F\left(\widehat{w_N^1}\left(p_0, p_r\right)\right)\right]$ and $(1-s)\left[F\left(\widehat{w_N^1}\left(p_0, p_r\right)\right) - F\left(\widehat{w_N^2}\left(p_r\right)\right)\right]$ respectively. As p_0 increases, the graph of U_N^B moves down. This lead to decrease of demand in the house market, but increase of it in the chonsei market. As p_r increases, the graph of U_N^L moves down. This lead to decrease of demand in the chonsei market, but increase of it in the house market. Thus the law of demand holds in each market.

If we combine Lemmas 7 and 9, we can get a sufficient condition for the existence of chonsei lease arrangement. If we have $\overline{p_r}\left(p_0^I\right) > \underline{p_r}\left(p_0^I\right)$, there will be non-zero demand and supply in chonsei market. ■[Note that $\overline{p_r}\left(p_0\right)$ is decreasing while $\underline{p_r}\left(p_0\right)$ is increasing in p_0. Therefore if p_0^I is low enough, there exists chonsei market. We already discussed comparative statics result for p_0^I. Without affecting utility levels of different choices, p_0^I can go low as we increase s or change the income distribution function F. When housing stock s is given, more income growth expectation through the change of income distribution function F makes the chonsei lease agreement more likely.

Proposition 8 *There exist some economic environments* $(s, F(\cdot), w_1, p_1, r)$ *where chonsei lease arrangement is possible. Chonsei lease arrangement is likely when* p_0^I *is low enough.*]

By summarizing the above argument, we can define an equilibrium in a house market. We abuse the notation and still denote equilibrium house price as p_0^I.

Definition 2 *Equilibrium of house markets for both purchase and lease is a pair* $\left(p_0^I, p_r^I\right)$ *which satisfies*

i) house market clearing

$$sF\left(\widehat{w_O^2}\left(p_0^I, p_r^I\right)\right) = (1-s)\left[1 - F\left(\widehat{w_N^1}\left(p_0^I, p_r^I\right)\right)\right]$$

ii) chonsei market clearing

$$s\left[F\left(\widehat{w_O^1}\left(p_r^I\right)\right) - F\left(\widehat{w_O^2}\left(p_0^I, p_r^I\right)\right)\right] = (1-s)\left[F\left(\widehat{w_N^1}\left(p_0^I, p_r^I\right)\right) - F\left(\widehat{w_N^2}\left(p_r^I\right)\right)\right]$$

The effect of chonsei market on house price is ambiguous. As Fig 7 show, some landlords would have sold the house without chonsei lease. Thus chonsei lease will reduce supply in the house market. Likewise, some tenants would have bought the house. Demand in the house market is also reduced. Thus we cannot definitely say the direction of price change.

We can apply the same comparative static analysis when chonsei lease is also available. This result is summarized in the following proposition.

Proposition 9 *Equilibrium prices* p_0^I *and* p_r^I *are affected by the parameters of model as follows.*
i) If s *increases, both* p_0^I *and* p_r^I *decrease.*
ii) If p_1 *increases, the effect on* p_0^I *is not certain while* p_r^I *decreases.*
iii) If r *increases, both* p_0^I *and* p_r^I *weakly decrease.*
iv) If w_1 *increases, both* p_0^I *and* p_r^I *decrease.*
v) If income follows a distribution function G *which is first order stochastically dominated by* F*, both* p_0^I *and* p_r^I *decrease.*

Proof. See the appendix. ■

If house stocks are more abundant, both its price and chonsei deposit will decrease. Note that house market and chonsei lease market (house price and chonsei deposit) interact with each other. Specifically, the change in excess demand due to price change in house market is exactly offset by the change in excess demand in Chonsei lease market. It turns out that the change in house price and chonsei deposit is more dependant on the total mass of house owners wanting to lease or sell and non house owners wanting to rent or buy rather than their compositions. For example, if income in the period 0 decreases as in v), for consumption smoothing purpose, more house owners are willing to lease or sell their houses while less non house owners are willing to rent or buy. Thus both house price and chonsei deposit will fall. The same logic applies if period 1 income, w_1 increases in iv). If interest rate increases in iii), the saving incentive is usually determined by the relative size of income and substitution effect. But a house owner at the margin who is indifferent between selling or leasing a house for example, is credit constrained when leasing a house. If this house owner sells a house instead, he or she may be able to enjoy the benefit of higher interest rate from saving. Thus more house owners want to lease or sell their houses while less non house owners want to rent or buy. As a result, both house price and chonsei deposit fall. If the future price of house increases in ii), house owners have more incentive to smooth consumption either through leasing or selling. Thus there is downward pressure on both house price and chonsei deposit. But to enjoy price appreciation, house owners are more likely to lease the house than sell. More non owners want to buy rather than rent it. Thus the chonsei deposit will fall but the effect on house price is ambiguous. The change in composition is likely to increase house price, but the fall in chonsei deposit may hinder its price increase.

4 Discussion

1. This paper's explanation of chonsei is different from existing literature in that chonsei is completely explained in the housing market itself. We do not assume excess return in other investment opportunities. The excess return of housing asset is not superimposed but endogenously explained in the model.

2. The key ingredient of chonsei phenomena is combination of fast income growth and credit constraint. Fast income growth foreshadow fast price increase of housing asset. But credit constraint does not allow the house price appreciation representing this expectation. Credit

constraint plays two key roles for the existence of chonsei market. First, it makes the house price undervalued and creates excess return of housing asset, which can be shared through chonsei contract. Second, chonsei lease can be a tool to share this excess return due to credit constraint. Landlords can accomplish consumption smoothing from up-front deposit and are willing to pay high interest. Thus some excess return will go to tenants by cheap chonsei deposit.

3. Widespread chonsei lease arrangement may mean expectation of high house price inflation. Most of literature pay attention to relationship between house price increase and chonsei deposit - house price ratio. However, even if we replace chonsei deposit with monthly rent, the same relationship holds. This paper argues that the share of chonsei lease arrangement may also have a relationship with the expectation of house price inflation. Recent withering of chonsei lease arrangement can be related to development of mortgage lending and exhaustion of excess return of housing asset. This is more formally argued in the extension of the model where the extent of credit constraint can be modified.

4. The possibility of excess return of an asset may not be restricted housing asset. Any asset or investment opportunity which requires sizable funding may suffer from credit constraint. Therefore, those asset classes may have low valuation considering their fundamentals and have excess return.

5 Extension

5.1 Another investment strategy

In our model, landlords use chonsei deposit for consumption smoothing purpose. However, as mentioned in the introduction, it is widely perceived that landlord mostly use chonsei lease deposit to purchase another house. That is, chonsei deposit is used as a leverage for another house investment. Our model does not allow this by assuming that an agent can own only one house.

In this subsection, we consider another investment strategy which can consider this aspect of chonsei deposit while maintaining our assumption. Non house owners can purchase a house and lease it through chonsei. Likewise, house owners can sell their houses and rent one through chonsei. We will see how this investment option can affect house markets.

[To be added].

5.2 Different types of credit constraint

There can be many types of financial market imperfection. Our model assume one extreme form that there is no borrowing in the financial market. In this subsection, we relax that assumption. We will consider two types of credit constraint. First, borrowing is possible but limited to a certain ratio of house price if one has house, which we call loan to value ratio credit constrain or LTV. Second, borrowing is limited to certain multiple to one's current income, which is called debt to income ratio credit constraint or DTI.

5.2.1 Loan to Value ratio credit constraint (LTV)

Under Loan to Value ratio credit constraint (LTV), lending requires housing asset as collateral and only a certain portion of house price can be borrowed. Let ν be the portion of house price p_0 on which lending can be made. The possibility of borrowing does not affect the present value of wealth, but affects the available period 0 income of those who currently possess houses. That is, house owners who keep or lease it thorough chonsei or non-house owners who newly purchase houses can increase their period 0 available income by up to νp_0. Moreover, chonsei deposit p_r will be bigger than $H + \nu p_0$. From the main analysis, we understand chonsei lease will lower the wealth level of house owners than just keeping a house. If $H + \nu p_0 \geq p_r$, a house owner would not lease a house through chonsei as keeping a house can secure as much period 0 available income as leasing. Therefore, compared with the main analysis without any borrowing, LTV credit constraint will weakly the utility level of keeping a house for house owners, U_O^0 and that of buying a house for non house owners, U_N^B as follows.

$$
\begin{aligned}
U_O^0(w_0, p_0) &= \begin{cases} u\left(c_0^O\right) + \beta u\left(c_1^O\right) \text{ if } w_0 + H + \nu p_0 \geq c_0^O \\ u(w_0 + H + \nu p_0) + \beta u\left(w_1 + p_1 - (1+r)\,\nu p_0\right) \text{ if } w_0 + H + \nu p_0 < c_0^O \end{cases} \\
U_N^B(w_0, p_0) &= \begin{cases} u\left(c_0^{NB}\right) + \beta u\left(c_1^{NB}\right) \text{ if } w_0 - (1-\nu)\,p_0 + H \geq c_0^{NB} \\ u(w_0 - (1-\nu)\,p_0 + H) + \beta u\left(w_1 + p_1 - (1+r)\,\nu p_0\right) \text{ if } w_0 - (1-\nu)\,p_0 + H < c_0^{NB} \end{cases}
\end{aligned}
$$

Except for these changes, the house owners' and non-house owners' decision are the same as depicted in Fig 7 and Fig 8. Thus, taking house owners for example, there are two threshold income levels and three intervals of income. Those in bottom income interval sell houses, those in the middle lease houses and those in the top keep houses.

We first note that the relaxed credit constraint make the existence of chonsei lease market less likely. Recall the condition for the existence of chonsei lease market, $\overline{p_r}(p_0) > \underline{p_r}(p_0)$. With LTV credit constraint, compared with no borrowing in the main analysis, $\underline{p_r}(p_0)$ will increase while $\overline{p_r}(p_0)$ decreases. Thus, the trade in Chonsei lease market is less likely. Moreover, given the current house price p_0, higher ν increases $\underline{p_r}(p_0)$ while decreasing $\overline{p_r}(p_0)$. With relaxed credit constraint, house owners can borrow on their houses rather than leasing them at low deposit. Thus they would ask for higher chonsei deposit. Non-house owners may well buy houses and borrow on them rather than renting them at high deposit. Thus they would ask for lower deposit. Then chonsei lease may not sustain.

If Chonsei lease market exists, we have similar equilibrium condition as in Definition 2. The threshold income level $\widehat{w_N^1}(p_0^I, p_r^I)$ and $\widehat{w_O^1}(p_0^I, p_r^I)$ would change, but we abuse the notation. We write the equilibrium condition as

$$\begin{aligned} sF\left(\widehat{w_O^2}(p_0^I, p_r^I)\right) &= (1-s)\left[1 - F\left(\widehat{w_N^1}(p_0^I, p_r^I)\right)\right] \\ s\left[F\left(\widehat{w_O^1}(p_0^I, p_r^I)\right) - F\left(\widehat{w_O^2}(p_0^I, p_r^I)\right)\right] &= (1-s)\left[F\left(\widehat{w_N^1}(p_0^I, p_r^I)\right) - F\left(\widehat{w_N^2}(p_r^I)\right)\right] \end{aligned}$$

The notable difference is that owner's threshold income level $\widehat{w_O^1}$ is also dependent on house price p_0. In Fig 7, if house price increases, house owners who keep the houses can borrow more and can achieve more consumption smoothing. Thus U_O^0 increases, and $\widehat{w_O^1}$ decreases. This difference makes a slight change in the comparative statics result which is summarized in the following proposition.

Proposition 10 *Equilibrium prices p_0^I and p_r^I are affected by the parameters of model as follows when equilibrium prices are stable.*

i) If s increases, both p_0^I and p_r^I decrease.

ii) If p_1 increases, the effect on p_0^I and p_r^I is not certain.

iii) If r increases, both p_0^I and p_r^I decreases.

iv) If w_1 increases, both p_0^I and p_r^I decrease.

v) If income follows a distribution function G which is first order stochastically dominated by F, both p_0^I and p_r^I decrease.

vi) If ν increases, both p_0^I and p_r^I increase.

Proof. See the appendix. ■

As shown in the proof, equilibrium price may not be stable when we have LTV credit constraint. The change in house price p_0 have more sizable effect on Chonsei lease market than on house market. The increase in p_0 reduces excess demand in house market which are offset by increase of excess demand in Chonsei lease market. There will be further increase of excess demand in Chonsei lease market as some house owners who can borrow more on their houses will not lease their houses any more. Therefore, some equilibrium prices may not be stable. As long as equilibrium prices are stable, most of the result carries over from Proposition 9 except for ii) and vi). The intuition behind the result is virtually the same.

The increase in future house price p_1 has two countervailing effect. House owners have more consumption smoothing incentive, but also want to keep the house to enjoy the future price. Thus, Chonsei deposit decreases while its effect on house price is uncertain in our main analysis. If house price happens to increase, house owners can borrow more on that and thus decrease the supply of Chonsei lease market. Therefore, the effect on Chonsei deposit also becomes ambiguous.

If LTV ratio ν increases, more house owners want to keep the house rather than leasing it while more non house owners want to buy a house. Thus both house price and Chonsei deposit will increase. Thus the tightening the LTV credit constraint does not only decrease house price but also constrain Chonsei deposit.

5.2.2 Debt to Income ratio credit constraint (DTI)

Under debt to income ratio credit constraint, debt is restricted so that debt service (interest and principal payment) relative to income is limited. In our model, that simply means a constant portion of period 0 income can be borrowed. Let μ be that constant portion. Whether one has a house or not, an agent borrow up to μw_0. This will weakly increase utility levels of all the decisions depending on the first period income w_0. For example, U_N^0, utility of non-house owners who neither rent nor buy a house, is changed into

$$U_N^0(w_0) = \begin{cases} u\left(c_0^N\right) + \beta u\left(c_1^N\right) \text{ if } (1+\mu)\, w_0 \geq c_0^N \\ u((1+\mu)\, w_0) + \beta u\left(w_1 - (1+r)\, \mu w_0\right) \text{ if } (1+\mu)\, w_0 < c_0^N \end{cases},$$

which is weakly greater than the utility level in main analysis.

However, the structure of decision is the same and the equilibrium condition is similar to main analysis. The result of the comparative statics analysis are similar except for the effect of interest rate.

Proposition 11 *Equilibrium prices* p_0^I *and* p_r^I *are affected by the parameters of model as follows.*
i) If s *increases, both* p_0^I *and* p_r^I *decrease.*
ii) If p_1 *increases, the effect on* p_0^I *is not certain while* p_r^I *decreases.*
iii) If r *increases, both* p_0^I *and* p_r^I *decreases.*
iv) If w_1 *increases, both* p_0^I *and* p_r^I *decrease.*
v) If income follows a distribution function G *which is first order stochastically dominated by* F*, both* p_0^I *and* p_r^I *decrease.*
vi) If μ *increases, both* p_0^I *and* p_r^I *increase.*

The effect of interest rate change becomes ambiguous. Consider a house owner who is indifferent between selling and leasing a house. If this house owner leases a house rather than sells it, he can still be indebted and will suffer from the negative impact of interest rate increase. We cannot exclude the possibility that the decrease of utility when selling a house is greater than that when leasing a house. Marginal house owners may want to lease rather than sell houses. By the same token, they may want to keep rather than lease houses. Thus it is possible that both house price and chonsei deposit increase.

6 Conclusion

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Appendix

Proof of Proposition 5 i) is trivial. Change in distribution function will affect supply and demand without any change of threshold income, and v) follows. Increase in p_1 increases U_O^0 and U_N^B and thus increases the supply and decreases the demand in housing market and thus iii) follows.

At $\widehat{w_O}(p_0)$, we have either

$$u\left(\widehat{w_O}(p_0) + H\right) + \beta u\left(w_1 + p_1\right) = u\left(c_0^{OS}\right) + \beta u\left(c_1^{OS}\right)$$

or

$$u\left(\widehat{w_O}(p_0) + H\right) + \beta u\left(w_1 + p_1\right) = u\left(\widehat{w_O}(p_0) + p_0\right) + \beta u\left(w_1\right).$$

Increase in r does not change $\widehat{w_O}(p_0)$ in the second case, but increases $\widehat{w_O}(p_0)$ in the first case. Agents are net savers when consumption path $\left(c_0^{OS}, c_1^{OS}\right)$ is chosen and interest rate increase means the expansion of budget set. Thus increase in r weakly increases the supply. Increase in w_1 will increase $\widehat{w_O}(p_0)$ as the RHS increases more than LHS in both cases since $w_1 + p_1$ is greater than c_1^{OS} or w_1. Thus increase in w_1 increases the supply.

Similarly, at $\widehat{w_N}(p_0)$, we have either

$$u(\widehat{w_N}(p_0) - p_0 + H) + \beta u(w_1 + p_1) = u\left(c_0^N\right) + \beta u\left(c_1^N\right)$$

or

$$u(\widehat{w_N}(p_0) - p_0 + H) + \beta u(w_1 + p_1) = u(\widehat{w_N}(p_0)) + \beta u(w_1).$$

Increase in r does not change $\widehat{w_N}(p_0)$ in the second case, but increase $\widehat{w_N}(p_0)$ in the first case. Thus increase in r weakly decreases the demand. Increase in w_1 will increases $\widehat{w_N}(p_0)$ as the RHS increases more than LHS in both cases since $w_1 + p_1$ is greater than c_1^N or w_1. Thus increase in w_1 decreases the demand.

Combining both changes, p_0^I decreases if r increases or w_1 increases.

Proof of Proposition 9 The result of comparative statics is obtained from equilibrium conditions through implicit function theorem. Let two equilibrium conditions be written as excess demands of two markets are equal to 0,

$$\begin{aligned}
\Psi_1 &= (1-s)\left[1 - F\left(\widehat{w_N^1}\left(p_0^I, p_r^I\right)\right)\right] - sF\left(\widehat{w_O^2}\left(p_0^I, p_r^I\right)\right) = 0 \\
\Psi_2 &= (1-s)\left[F\left(\widehat{w_N^1}\left(p_0^I, p_r^I\right)\right) - F\left(\widehat{w_N^2}\left(p_r^I\right)\right)\right] - s\left[F\left(\widehat{w_O^1}\left(p_r^I\right)\right) - F\left(\widehat{w_O^2}\left(p_0^I, p_r^I\right)\right)\right] = 0.
\end{aligned}$$

Given any parameter δ, we have

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial p_0^I}\frac{\partial p_0^I}{\partial \delta} + \frac{\partial \Psi_1}{\partial p_r^I}\frac{\partial p_r^I}{\partial \delta} &= -\frac{\partial \Psi_1}{\partial \delta} \\
\frac{\partial \Psi_2}{\partial p_0^I}\frac{\partial p_0^I}{\partial \delta} + \frac{\partial \Psi_2}{\partial p_r^I}\frac{\partial p_r^I}{\partial \delta} &= -\frac{\partial \Psi_2}{\partial \delta}
\end{aligned}$$

or

$$\begin{bmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & \frac{\partial \Psi_1}{\partial p_r^I} \\ \frac{\partial \Psi_2}{\partial p_0^I} & \frac{\partial \Psi_2}{\partial p_r^I} \end{bmatrix} \begin{bmatrix} \frac{\partial p_0^I}{\partial \delta} \\ \frac{\partial p_r^I}{\partial \delta} \end{bmatrix} = \begin{bmatrix} -\frac{\partial \Psi_1}{\partial \delta} \\ -\frac{\partial \Psi_2}{\partial \delta} \end{bmatrix}.$$

Using Cramer's rule, we have

$$\frac{\partial p_0^I}{\partial \delta}=\frac{\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \delta} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \delta} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & \frac{\partial \Psi_1}{\partial p_r^I} \\ \frac{\partial \Psi_2}{\partial p_0^I} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}},\ \frac{\partial p_r^I}{\partial \delta}=\frac{\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \delta} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \delta} \end{vmatrix}}{\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & \frac{\partial \Psi_1}{\partial p_r^I} \\ \frac{\partial \Psi_2}{\partial p_0^I} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}}.$$

Since law of supply and demand holds in each market and sales and leases of houses are substitutes (as $\frac{\partial \widehat{w_O^2}}{\partial p_0^I}, \frac{\partial \widehat{w_N^1}}{\partial p_0^I}, \frac{\partial \widehat{w_O^1}}{\partial p_r^I}, \frac{\partial \widehat{w_N^2}}{\partial p_r^I}>0$ and $\frac{\partial \widehat{w_O^2}}{\partial p_r^I}, \frac{\partial \widehat{w_N^1}}{\partial p_r^I}<0$), we have

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial p_0^I} &= -\left\{(1-s)\, f\left(\widehat{w_N^1}\right)\frac{\partial \widehat{w_N^1}}{\partial p_0^I}+sf\left(\widehat{w_O^2}\right)\frac{\partial \widehat{w_O^2}}{\partial p_0^I}\right\}<0\\
\frac{\partial \Psi_2}{\partial p_0^I} &= -\frac{\partial \Psi_1}{\partial p_0^I}>0\\
\frac{\partial \Psi_1}{\partial p_r^I} &= -\left\{(1-s)\, f\left(\widehat{w_N^1}\right)\frac{\partial \widehat{w_N^1}}{\partial p_r^I}+sf\left(\widehat{w_O^2}\right)\frac{\partial \widehat{w_O^2}}{\partial p_r^I}\right\}>0\\
\frac{\partial \Psi_2}{\partial p_r^I} &= -\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I}+sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}-\frac{\partial \Psi_1}{\partial p_r^I}<0.
\end{aligned}$$

Moreover, if we postulate that price increases (or decreases) when excess demand is positive (or negative), then equilibrium prices are stable as the derivative of excess demand is negative definite. That is, $\frac{\partial \Psi_1}{\partial p_0^I}<0$ and

$$\begin{aligned}
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & \frac{\partial \Psi_1}{\partial p_r^I} \\ \frac{\partial \Psi_2}{\partial p_0^I} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I}\left[-\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I}+sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}-\frac{\partial \Psi_1}{\partial p_r^I}\right]+\frac{\partial \Psi_1}{\partial p_0^I}\frac{\partial \Psi_1}{\partial p_r^I}\\
&= -\frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I}+sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\}>0.
\end{aligned}$$

Thus the comparative statics are determined by the sign of $\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \delta} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \delta} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix}$ and $\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \delta} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \delta} \end{vmatrix}$.

For i), as

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial s} &= -\left\{\left[1-F\left(\widehat{w_N^1}\right)\right]+F\left(\widehat{w_O^2}\right)\right\}<0\\
\frac{\partial \Psi_2}{\partial s} &= -\left[F\left(\widehat{w_N^1}\right)-F\left(\widehat{w_N^2}\right)\right]-\left[F\left(\widehat{w_O^1}\right)-F\left(\widehat{w_O^2}\right)\right]<0,
\end{aligned}$$

we have $\frac{\partial p_0^I}{\partial s}, \frac{\partial p_r^I}{\partial s} < 0$.

For v), let us assume another parameter ε such that $\frac{\partial F}{\partial \varepsilon} > 0$. Then the distribution with higher ε is first order stochastically dominated by the one with lower ε. As

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial \varepsilon} &= -(1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^1}\right) - s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^2}\right) < 0 \\
\frac{\partial \Psi_2}{\partial \varepsilon} &= -\frac{\partial \Psi_1}{\partial \varepsilon} - (1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^2}\right) - s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^1}\right),
\end{aligned}$$

we have

$$\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \varepsilon} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \varepsilon} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial \varepsilon}\left\{(1-s) f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\} - \frac{\partial \Psi_1}{\partial p_r^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^2}\right) + s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^1}\right)\right\} \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \varepsilon} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \varepsilon} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^2}\right) + s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^1}\right)\right\} < 0.
\end{aligned}$$

Thus $\frac{\partial p_0^I}{\partial \varepsilon}, \frac{\partial p_r^I}{\partial \varepsilon} < 0$.

For other comparative statics, we first need to think of the change in the threshold income levels; $\widehat{w_O^1}$, $\widehat{w_O^2}$, $\widehat{w_N^1}$, and $\widehat{w_N^2}$. At $\widehat{w_O^1}$, for example, we have either

$$u(\widehat{w_O^1} + H) + \beta u(w_1 + p_1) = u\left(c_0^{OL}\right) + \beta u\left(c_1^{OL}\right)$$

or

$$u(\widehat{w_O^1} + H) + \beta u(w_1 + p_1) = u(w_0 + p_r) + \beta u(w_1 + p_1 - p_r).$$

When p_1 increases, RHS increases more than LHS as $w_1 + p_1 > c_1^{OL}$(or $w_1 + p_1 - p_r$). Thus $\frac{\partial \widehat{w_O^1}}{\partial p_1} > 0$. When r increases, RHS in the first case increases as agents are net savers when $\left(c_0^{OL}, c_1^{OL}\right)$ is chosen. In the second case, it does not affect either LHS or RHS. Thus $\frac{\partial \widehat{w_O^1}}{\partial r} \geq 0$. When w_1 increases, RHS increases more than LHS with the same reason above, $\frac{\partial \widehat{w_O^1}}{\partial w_1} > 0$.

We can do similar exercise for other threshold incomes: $\widehat{w_O^2}$, $\widehat{w_N^1}$, and $\widehat{w_N^2}$. For change in p_1, we have $\frac{\partial \widehat{w_O^2}}{\partial p_1} < 0$, $\frac{\partial \widehat{w_N^1}}{\partial p_1} < 0$, and $\frac{\partial \widehat{w_N^2}}{\partial p_1} = 0$. For change in r, we have $\frac{\partial \widehat{w_O^2}}{\partial r} \geq 0$, $\frac{\partial \widehat{w_N^1}}{\partial r} \geq 0$, and $\frac{\partial \widehat{w_N^2}}{\partial r} \geq 0$. For change in w_1, $\frac{\partial \widehat{w_O^2}}{\partial w_1} > 0$, $\frac{\partial \widehat{w_N^1}}{\partial w_1} > 0$, and $\frac{\partial \widehat{w_N^2}}{\partial w_1} > 0$.

For ii), as

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial p_1} &= -(1-s)\, f\left(\widehat{w_N^1}\right) \frac{\partial \widehat{w_N^1}}{\partial p_1} - sf\left(\widehat{w_O^2}\right) \frac{\partial \widehat{w_O^2}}{\partial p_1} > 0 \\
\frac{\partial \Psi_2}{\partial p_1} &= -sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_1} - \frac{\partial \Psi_1}{\partial p_1} < 0,
\end{aligned}$$

we have

$$\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial p_1} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial p_1} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_1} \left\{ sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_r^I} + (1-s)\, f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial p_r^I} \right\} - \frac{\partial \Psi_1}{\partial p_r^I} sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_1} \lesseqgtr 0 \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial p_1} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial p_1} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I} sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_1} < 0.
\end{aligned}$$

For iii), as

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial r} &= -(1-s)\, f\left(\widehat{w_N^1}\right) \frac{\partial \widehat{w_N^1}}{\partial r} - sf\left(\widehat{w_O^2}\right) \frac{\partial \widehat{w_O^2}}{\partial r} \leq 0 \\
\frac{\partial \Psi_2}{\partial r} &= -(1-s)\, f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial r} - sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial r} - \frac{\partial \Psi_1}{\partial r} \lesseqgtr 0,
\end{aligned}$$

we have

$$\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial r} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial r} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial r} \left\{ (1-s)\, f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_r^I} \right\} - \frac{\partial \Psi_1}{\partial p_r^I} \left\{ (1-s)\, f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial r} + sf\left(\widehat{w_O^1}\right) \right. \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial r} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial r} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I} \left\{ (1-s)\, f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial r} + sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial r} \right\} \leq 0.
\end{aligned}$$

For iv), as

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial w_1} &= -(1-s)\, f\left(\widehat{w_N^1}\right) \frac{\partial \widehat{w_N^1}}{\partial w_1} - sf\left(\widehat{w_O^2}\right) \frac{\partial \widehat{w_O^2}}{\partial w_1} < 0 \\
\frac{\partial \Psi_2}{\partial w_1} &= -(1-s)\, f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial w_1} - sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial w_1} - \frac{\partial \Psi_1}{\partial w_1} \lesseqgtr 0,
\end{aligned}$$

we have

$$\begin{vmatrix} -\frac{\partial\Psi_1}{\partial w_1} & \frac{\partial\Psi_1}{\partial p_r^I} \\ -\frac{\partial\Psi_2}{\partial w_1} & \frac{\partial\Psi_2}{\partial p_r^I}\end{vmatrix} = \frac{\partial\Psi_1}{\partial w_1}\left\{(1-s) f\left(\widehat{w_N^2}\right)\frac{\partial\widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\} - \frac{\partial\Psi_1}{\partial p_r^I}\left\{(1-s) f\left(\widehat{w_N^2}\right)\frac{\partial\widehat{w_N^2}}{\partial w_1} + sf\left(\widehat{w_O^1}\right) \cdots\right.$$

$$\begin{vmatrix} \frac{\partial\Psi_1}{\partial p_0^I} & -\frac{\partial\Psi_1}{\partial w_1} \\ \frac{\partial\Psi_2}{\partial p_0^I} & -\frac{\partial\Psi_2}{\partial w_1}\end{vmatrix} = \frac{\partial\Psi_1}{\partial p_0^I}\left\{(1-s) f\left(\widehat{w_N^2}\right)\frac{\partial\widehat{w_N^2}}{\partial w_1} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial w_1}\right\} < 0.$$

Proof of Proposition 10 The analysis is basically the same as in Proposition 9 except that the equilibrium conditions are slightly changed,

$$\begin{aligned}
\Psi_1 &= (1-s)\left[1-F\left(\widehat{w_N^1}\left(p_0^I,p_r^I\right)\right)\right] - sF\left(\widehat{w_O^2}\left(p_0^I,p_r^I\right)\right) = 0 \\
\Psi_2 &= (1-s)\left[F\left(\widehat{w_N^1}\left(p_0^I,p_r^I\right)\right) - F\left(\widehat{w_N^2}\left(p_r^I\right)\right)\right] - s\left[F\left(\widehat{w_O^1}\left(p_0^I,p_r^I\right)\right) - F\left(\widehat{w_O^2}\left(p_0^I,p_r^I\right)\right)\right] = 0.
\end{aligned}$$

We abuse the notation and keep the same notation as before. Note that the same notation does not mean the same function. Still law of supply and demand holds in each market and sales and leases of houses are substitutes. We additionally have $\frac{\partial\widehat{w_O^1}}{\partial p_0^I} < 0$ in addition to $\frac{\partial\widehat{w_O^2}}{\partial p_0^I}, \frac{\partial\widehat{w_N^1}}{\partial p_0^I}, \frac{\partial\widehat{w_O^1}}{\partial p_r^I}, \frac{\partial\widehat{w_N^2}}{\partial p_r^I} > 0$ and $\frac{\partial\widehat{w_O^2}}{\partial p_r^I}, \frac{\partial\widehat{w_N^1}}{\partial p_r^I} < 0$. The expression $\frac{\partial\Psi_1}{\partial p_0^I}$, $\frac{\partial\Psi_1}{\partial p_r^I}$, and $\frac{\partial\Psi_2}{\partial p_r^I}$ are the same as in the proof of Proposition 9, and $\frac{\partial\Psi_2}{\partial p_0^I}$ is changed to

$$\frac{\partial\Psi_2}{\partial p_0^I} = -\frac{\partial\Psi_1}{\partial p_0^I} - sf\left(\widehat{w_O^1}\right)\frac{\partial\widehat{w_O^1}}{\partial p_0^I} > 0.$$

Equilibrium prices are not guaranteed to be stable as the derivative of excess demand is not necessarily negative definite. That is,

$$\begin{vmatrix} \frac{\partial\Psi_1}{\partial p_0^I} & \frac{\partial\Psi_1}{\partial p_r^I} \\ \frac{\partial\Psi_2}{\partial p_0^I} & \frac{\partial\Psi_2}{\partial p_r^I}\end{vmatrix} = -\frac{\partial\Psi_1}{\partial p_0^I}\left\{(1-s) f\left(\widehat{w_N^2}\right)\frac{\partial\widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\} + \frac{\partial\Psi_1}{\partial p_r^I} sf\left(\widehat{w_O^1}\right)\frac{\partial\widehat{w_O^1}}{\partial p_0^I} \lesseqgtr 0$$

which can be negative. If we restrict our attention to stable equilibrium prices, the comparative statics are determined by the sign of numerator in Cramer's rule we use.

For i), as

$$\begin{aligned}
\frac{\partial\Psi_1}{\partial s} &= -\left\{\left[1-F\left(\widehat{w_N^1}\right)\right] + F\left(\widehat{w_O^2}\right)\right\} < 0 \\
\frac{\partial\Psi_2}{\partial s} &= -\left[F\left(\widehat{w_N^1}\right) - F\left(\widehat{w_N^2}\right)\right] - \left[F\left(\widehat{w_O^1}\right) - F\left(\widehat{w_O^2}\right)\right] < 0,
\end{aligned}$$

we have $\frac{\partial p_0^I}{\partial s}, \frac{\partial p_r^I}{\partial s} < 0$.

For v), let us assume another parameter ε such that $\frac{\partial F}{\partial \varepsilon} > 0$. Then the distribution with higher ε is first order stochastically dominated by the one with lower ε. As

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial \varepsilon} &= -(1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^1}\right) - s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^2}\right) < 0 \\
\frac{\partial \Psi_2}{\partial \varepsilon} &= -\frac{\partial \Psi_1}{\partial \varepsilon} - (1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^2}\right) - s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^1}\right),
\end{aligned}$$

we have

$$\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \varepsilon} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \varepsilon} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial \varepsilon}\left\{(1-s) f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\} - \frac{\partial \Psi_1}{\partial p_r^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^2}\right) + s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^1}\right)\right\} \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \varepsilon} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \varepsilon} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_N^2}\right) + s\frac{\partial F}{\partial \varepsilon}\left(\widehat{w_O^1}\right)\right\} + \frac{\partial \Psi_1}{\partial \varepsilon} sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_0^I} < 0.
\end{aligned}$$

Thus $\frac{\partial p_0^I}{\partial \varepsilon}, \frac{\partial p_r^I}{\partial \varepsilon} < 0$.

For other comparative statics, we need to think of the change in the threshold income levels; $\widehat{w_O^1}$, $\widehat{w_O^2}$, $\widehat{w_N^1}$, and $\widehat{w_N^2}$. We have

$$\begin{aligned}
\frac{\partial \widehat{w_O^1}}{\partial p_1} &> 0, \frac{\partial \widehat{w_O^2}}{\partial p_1} < 0, \frac{\partial \widehat{w_N^1}}{\partial p_1} < 0, \frac{\partial \widehat{w_N^2}}{\partial p_1} = 0 \\
\frac{\partial \widehat{w_O^1}}{\partial w_1} &> 0, \frac{\partial \widehat{w_O^2}}{\partial w_1} > 0, \frac{\partial \widehat{w_N^1}}{\partial w_1} > 0, \frac{\partial \widehat{w_N^2}}{\partial w_1} > 0 \\
\frac{\partial \widehat{w_O^1}}{\partial r} &> 0, \frac{\partial \widehat{w_O^2}}{\partial r} \geq 0, \frac{\partial \widehat{w_N^1}}{\partial r} > 0, \frac{\partial \widehat{w_N^2}}{\partial r} \geq 0 \\
\frac{\partial \widehat{w_O^1}}{\partial \nu} &< 0, \frac{\partial \widehat{w_O^2}}{\partial \nu} = 0, \frac{\partial \widehat{w_N^1}}{\partial \nu} < 0, \frac{\partial \widehat{w_N^2}}{\partial \nu} = 0.
\end{aligned}$$

Analysis is almost similar for other parameters. For increase in ν, it will increase the utility of house owner when keeping the house and that of non house owners when buying the house.

For ii), as

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial p_1} &= -(1-s) f\left(\widehat{w_N^1}\right)\frac{\partial \widehat{w_N^1}}{\partial p_1} - sf\left(\widehat{w_O^2}\right)\frac{\partial \widehat{w_O^2}}{\partial p_1} > 0 \\
\frac{\partial \Psi_2}{\partial p_1} &= -sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_1} - \frac{\partial \Psi_1}{\partial p_1} < 0,
\end{aligned}$$

we have

$$
\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial p_1} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial p_1} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_1}\left\{ sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I} + (1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I}\right\} - \frac{\partial \Psi_1}{\partial p_r^I} sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_1} \lesseqgtr 0 \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial p_1} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial p_1} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I} sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_1} - \frac{\partial \Psi_1}{\partial p_1} sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_0^I} \lesseqgtr 0.
\end{aligned}
$$

For iii), as

$$
\begin{aligned}
\frac{\partial \Psi_1}{\partial r} &= -(1-s)\, f\left(\widehat{w_N^1}\right)\frac{\partial \widehat{w_N^1}}{\partial r} - sf\left(\widehat{w_O^2}\right)\frac{\partial \widehat{w_O^2}}{\partial r} < 0 \\
\frac{\partial \Psi_2}{\partial r} &= -(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial r} - sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial r} - \frac{\partial \Psi_1}{\partial r} \lesseqgtr 0,
\end{aligned}
$$

we have

$$
\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial r} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial r} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial r}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\} - \frac{\partial \Psi_1}{\partial p_r^I}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial r} + sf\left(\widehat{w_O^1}\right) \right. \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial r} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial r} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial r} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial r}\right\} - \frac{\partial \Psi_1}{\partial r} sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial r} < 0.
\end{aligned}
$$

For iv), as

$$
\begin{aligned}
\frac{\partial \Psi_1}{\partial w_1} &= -(1-s)\, f\left(\widehat{w_N^1}\right)\frac{\partial \widehat{w_N^1}}{\partial w_1} - sf\left(\widehat{w_O^2}\right)\frac{\partial \widehat{w_O^2}}{\partial w_1} < 0 \\
\frac{\partial \Psi_2}{\partial w_1} &= -(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial w_1} - sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial w_1} - \frac{\partial \Psi_1}{\partial w_1} \lesseqgtr 0,
\end{aligned}
$$

we have

$$
\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial w_1} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial w_1} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial w_1}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_r^I}\right\} - \frac{\partial \Psi_1}{\partial p_r^I}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial w_1} + sf\left(\widehat{w_O^1}\right) \right. \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial w_1} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial w_1} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I}\left\{(1-s)\, f\left(\widehat{w_N^2}\right)\frac{\partial \widehat{w_N^2}}{\partial w_1} + sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial w_1}\right\} - \frac{\partial \Psi_1}{\partial w_1} sf\left(\widehat{w_O^1}\right)\frac{\partial \widehat{w_O^1}}{\partial p_0^I} < 0.
\end{aligned}
$$

For vi), as

$$\begin{aligned}
\frac{\partial \Psi_1}{\partial \nu} &= -(1-s) f\left(\widehat{w_N^1}\right) \frac{\partial \widehat{w_N^1}}{\partial w_1} > 0 \\
\frac{\partial \Psi_2}{\partial \nu} &= -sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial \nu} - \frac{\partial \Psi_1}{\partial \nu} \lesseqgtr 0,
\end{aligned}$$

we have

$$\begin{aligned}
\begin{vmatrix} -\frac{\partial \Psi_1}{\partial \nu} & \frac{\partial \Psi_1}{\partial p_r^I} \\ -\frac{\partial \Psi_2}{\partial \nu} & \frac{\partial \Psi_2}{\partial p_r^I} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial \nu} \left\{ (1-s) f\left(\widehat{w_N^2}\right) \frac{\partial \widehat{w_N^2}}{\partial p_r^I} + sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_r^I} \right\} - \frac{\partial \Psi_1}{\partial p_r^I} sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial \nu} > 0 \\
\begin{vmatrix} \frac{\partial \Psi_1}{\partial p_0^I} & -\frac{\partial \Psi_1}{\partial \nu} \\ \frac{\partial \Psi_2}{\partial p_0^I} & -\frac{\partial \Psi_2}{\partial \nu} \end{vmatrix} &= \frac{\partial \Psi_1}{\partial p_0^I} sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial \nu} - \frac{\partial \Psi_1}{\partial \nu} sf\left(\widehat{w_O^1}\right) \frac{\partial \widehat{w_O^1}}{\partial p_0^I} > 0.
\end{aligned}$$