Incentives of Low-Quality Sellers to Voluntarily Disclose Negative Information*

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Abstract

In the paper we focus on incentives of low-quality sellers to disclose quality information. We study the role of two factors: buyers' uncertainty-aversion, modeled as either risk- or loss-aversion, and competition. Sellers can send a cheap talk message that buyers use to update their beliefs about product's quality. In the monopoly setting with risk-neutral buyers no relevant information is transmitted. When buyers are uncertainty-averse then low-quality, but never high-quality, sellers can choose to disclose the quality information. Disclosure of negative information benefits sellers as it increases uncertainty-averse buyers' willingness to pay. The duopoly setting provides additional incentives to disclose quality information as it creates product differentiation and softens the intensity of competition. We show that it allows for equilibira that would not be possible in the monopoly setting, for example, where cheap talk messages allow high-quality sellers to separate.

Keywords: Negative Information, product differentiation, cheap talk, lemon markets

JEL classification: D21, L15

1 Introduction

Is honesty the best policy for sellers? This well-known aphorism does not seem to be widely adopted in many markets as sellers often choose to conceal negative information about their products and services, especially when there exists information asymmetry between sellers and customers. The literature has also supported sellers? unwillingness to reveal negative information by documenting how negative information can damage sellers? performances. More specifically, many studies

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have shown that negative information decreases sales and purchase likelihood through various routes such as publicity (Tybout, Calder, & Sternthal, 1981; Wyatt & Badger, 1984), word-of-mouth (Arndt, 1967; Engel, Kegerreis, & Blackwell, 1969; Haywood, 1989; Laczniak, DeCarlo, & Ramaswami, 2001; Mizerski, 1982; Wright, 1974), and customer reviews (Basuroy, Chatterjee, & Ravid, 2003; Chevalier & Mayzlin, 2006; Clemons, Gao, & Hitt, 2006; Dellarocas, Zhang, & Awad, 2007; Reinstein & Snyder, 2005), suggesting the reason why incomplete disclosure can be frequently observed in many markets. Accordingly, the literature on information disclosure has focused on the tension between consumers who want more quality information and low-quality sellers who would like to hide it (Dranove & Jin, 2010).

What is interesting is that we can also often observe some sellers who voluntarily share negative aspects of their products and claim low quality. For example, many online retailers these days do not seem to be afraid of disclosing weaknesses about their products through various routes such as product descriptions or customer reviews on their own websites. Woot.com is especially famous for its preemptive revelation of the disadvantages of listed products, stating that they would prefer customers not buying from them to regretting their purchases. [1] A considerable number of sellers at many consumer-to-consumer online marketplaces such as eBay.com or Craiglist.org also voluntarily describe the weaknesses of their listed products and even show pictures of specific damages and scratches. For example, after purchasing collectible baseball cards from eBay and letting professional graders estimate true quality of those products, Jin & Kato (2006) have found that many sellers claiming lower grade were actually being honest although buyers cannot correctly evaluate the quality. Moreover, sellers at Craiglist's cars & trucks section often voluntarily reveal information about past accidents of their cars although there is no reputation tracking and customers cannot easily figure out the repair history of those used cars. In other words, these sellers at eBay and Craiglist simply reveal weaknesses of their products even when it hardly hurts to hide them.

Although above mentioned phenomenon of seller's honesty is not uncommon in many other markets, it has not received much attention in the academic literature so far. Rather, many studies have generally agreed that there exists incentive for high-quality sellers to reveal their types, but not for low-quality sellers, and that high-quality types always try to separate themselves from low-quality types by revealing relevant information (Akerlof, 1976; Milgrom, 1981; Spence, 1973). The literature has also suggested that reputation motive may alleviate the incomplete disclosure of low-type sellers since firms prefer to maintain their reputations and thus honestly share quality-related information under the possibility of repeated purchases (Farrell, 1980; Heal, 1976; Riordan, 1986; Shapiro, 1982, 1983; Smallwood & Conlisk, 1979; Wilson, 1985).

This paper thus aims to provide a new perspective on information disclosure in markets by analyzing the motivation of low-quality sellers to voluntarily reveal their types in a market with information asymmetry, even when reputation concern is negligible and customers cannot easily evaluate the quality ex post (i.e., experience or credence goods).

In our paper, we consider two factors that can incentivize low-quality sellers to reveal their qual-

[1] http://www.woot.com/faq?ref=ft_wiw_faq (accessed on November 14, 2015). Archived version: https://archive.is/b9cxI

ity. First, removing risk of associated with purchasing the product of unknown quality increases buyers' willingness to pay and, therefore, may provide incentive for sellers' voluntary information disclosure. It is will established in the literature that risk is a major influence on customer choice (Bauer, 1960; Dowling, 1986; Markin, Jr., 1974; Ross, 1975; Stone & Winter, 1985; Taylor, 1974), and purchasing decisions. For example, Dewally & Ederington (2006) have shown that risk reduction increases the valuation of listed products on online auctions and thus raises final prices. Second, competitive pressure can also work as an incentive for information disclosure. This is because a seller can differentiate his product from that of the competitor and soften the intensity of competition by revealing its quality.

As a benchmark, we first develop a model with a monopolistic seller and risk-neutral buyers. In this model, buyers do not observe product's quality but know the quality distribution. All buyers prefer higher quality products over lower quality.[2] The seller sends a cheap talk message that buyers use to update their beliefs about product's quality. Updated beliefs determine the demand function for the seller's product. The seller then sets the profit-maximizing price taking into account the demand for his product. When buyers are risk-neutral they only care about the expected quality and it is trivial to show that no information transmission about expected quality is possible. When a product's prior expected quality is Ev, for any on-equilibrium message posterior expected quality is also Ev. One cannot have two on-equilibrium messages with different expected qualities since all sellers strictly prefer the message with higher expected quality.

We then modify the framework by assuming that buyers are uncertainty-averse, i.e. they dislike uncertainty about product's quality. We will use two standard frameworks to capture uncertainty-aversion: risk-aversion and loss-aversion. We will use a generic term *uncertainty-aversion* when talking about both frameworks. Buyers differ in the strength of their uncertainty-aversion. Other things being equal, buyers with high (low) uncertainty-aversion are more (less) likely to purchase the product with low quality-uncertainty.

The model works in such a way that sellers of high-quality products — defined as those whose quality is above the average — cannot separate from low-quality sellers — those whose quality is below the average. Given focus on low-quality sellers, we design our model in such a way that it lacks standard tools that high-quality sellers can use to separate: the game is not repeated, there is no credible signaling device, certification is absent, and prices are now an effective signal for quality.[3] If there were an equilibrium cheap talk message that separates a high-quality seller then all low-quality sellers would prefer to send the same message as well. If it is possible to reveal that one's quality is above average and also eliminate uncertainty about quality, then sellers

[2] This is different from Tadelis and Zettelmeyer (2015) where some buyers prefer lower quality, while others prefer higher quality.

[3] Although theoretical literature has shown how price works as quality signal, empirical literature has presented that price is positively related with quality depending on the product category. For example, Rao & Monroe (1989) and Lichtenstein & Burton (1989) have found that customers are not capable of predicting quality with price signal if it is durable, higher-priced, non-frequently purchased product. Our paper also focuses on those non-frequently purchased products, for which price provides no predicting power. Section 5 relaxes this assumption and shows what happens if price works as quality signal.

might enjoy a high price by doing that. It would then be impossible to prevent a low-quality seller from mimicking a high-quality seller. Thus, if disclosure of information is possible in equilibrium then it should only be possible for low-quality sellers. We provide a characterization of possible equilibria and show that there are equilibria where sellers with quality below average can send a cheap talk message that reveals their quality. We also show that only one type can separate in equilibrium. Were it two types, the separating seller with lower quality would find it optimal to mimic a separating seller with a higher quality.

Everything said above holds for both loss- and risk-averse buyers. The main difference between the two frameworks is that under risk-aversion there is no equilibrium where the seller with the lowest quality separates. But such an equilibrium exists under loss-aversion. This difference will allow us to highlight the role of competition on incentives to disclose negative information. The reason why this difference exists is that the expected utility framework respects state-dominance while loss-aversion framework does not. In other words, no matter how risk-averse buyers are their willingness to pay for the product of the lowest quality, even if it is certain, is always less than their willingness to pay for the product whose quality can be lowest but has a positive chance of being higher. This is not true for loss-averse preferences. If a buyer is sufficiently loss-averse, then he can choose certain lowest quality just to avoid the possibility of loss from purchasing the product with quality below the expected.

In the second part of the paper we consider how incentives and opportunities to disclose quality information change in the competitive environment when there are two sellers. As one would expect, the combination of competition and uncertainty-averse buyers allows for more outcomes in the equilibria. There are some similarities between the monopoly and duopoly settings such as there is no information transmission in the case of risk-neutral buyers and one type can separate at most. However, there are notable differences as well. Specifically, in the duopoly setting, there are equilibria where highest-quality type separate, and in the case of risk-averse buyers, it is now possible for the lowest type to separate. The reason is that the competitive environment changes sellers' relative benefits from pooling and separating. This is because, unlike the monopoly case, the pooling equilibrium in the duopoly setting will result in Bertrand competition. If both sellers send the same message regardless of their quality then the products are identical from buyers' point of view and they will purchase the product with the lowest price. It is when buyers' have different beliefs about products of sellers i and j that it creates product differentiation and softens the competition.

We use a simple setting with two quality types to show that the benefits of product differentiation can outweigh the benefits of pooling with sellers of higher quality. To see when it happens, notice that whenever a low-quality seller chooses to separate oneself, or chooses not to mimic a separating high-quality type, he effectively sells the inferior product. In the former case its quality is known to be the lowest; in the second case not only its quality is known to be less than that of the separating high-quality sellers but also is uncertain. The only way then the low-quality seller can compete with the high-quality seller is if the number of buyers who can get attracted by the low-quality seller is small so that it is suboptimal for the high-quality seller to compete for them.

In the former case when the quality is known to be the lowest, such a seller has relatively more attractive products for buyers with high degree of uncertainty-aversion. Thus there must be sufficiently few buyers who are extremely uncertainty-averse so that it is suboptimal for high-quality seller to compete for them. In the latter case, the low-quality seller is relatively more attractive for buyers with low degree of uncertainty-aversion. Again, it means that there are sufficiently few buyers with low degree of uncertainty-aversion. The paper derives the exact conditions that specify what "sufficiently few buyers" mean in each case.

As long as low- and high-quality seller can co-exist and split the market one can get an equilibrium. In the equilibrium, the high-quality sellers earns very high profit if matched against the low-quality seller but the likelihood of this match is low. The likelihood of being matched with another high-quality seller is very high and whenever that happens the seller earns zero profit. For the low-quality seller, on the other hand, the low profit earned from the match with the high-quality competitor is compensated by a high likelihood of such match and low likelihood of being matched with another low-quality seller which would result in intense Bertrand competition.

Overall, this study attempts to provide a new perspective to the literature on information asymmetry in markets by re-examining the conventional wisdom about sellers' honesty. We show that low-quality sellers have incentives to reveal their quality due to the effect of risk on purchase. We believe that the analysis of the effect of risk in our paper provides important theoretical understanding about sellers and buyers' behavior under information asymmetry, and provide feasible explanations on why sellers voluntarily reveal weaknesses in certain situations. For example, many online retailers originally worried that revealing unfavorable reviews might have negative effects on their bottom line. However, according to Craig Berman, Amazon's vice president of global communications, those product reviews actually turned out to have positive effects on their performances despite those initial fears.[4] Ghose & Ipeirotis (2011) have also come up with evidences of these effects as they have analyzed online product reviews with text mining techniques and found that negative product reviews are associated with increased product sales when the review text is informative and detailed, thus providing sufficient information to customers. We believe that our study can provide theoretical explanation to this and other market phenomena where revealing low quality increases sales through reduced risk.

The paper is organized as follows. In Section 2 we review related literature. In Section 3 we introduce our benchmark model assuming monopoly, and in Section 4 we consider the effect on competition by examining the duopoly situation. Section 5 provides an extension of the model by showing what happens when price is used as signals of quality. We summarize and discuss the findings of this paper in Section 6. All proofs are in the appendix.

[4] Webley, K. (2010, July 16). A Brief History of Online Shopping. Time. Retrieved from http://content.time.com/time/business/article/0,8599,2004089,00.html

2 Literature Review

Ever since Akerlof (1970) has first demonstrated the possibility of market failures caused by information asymmetry, numerous studies have explained how information disclosure can mitigate adverse selections under information asymmetry. While the economic incentive of low-quality sellers to reveal their types or how the disclosure of negative information helps sellers have not been the center of related academic discussions, several studies in marketing and economics have provided some important understandings on this subject.

Some primary studies on information disclosure have argued that full disclosure naturally happens in markets with information asymmetry for the following reason. If a seller does not disclose the quality, then a buyer will assume that the quality is the lowest in the set of possible quality levels, since the seller could have made more profit by disclosing true quality if its quality is not the lowest in the subset. Therefore, the disclosure of information happens top down in an "unraveling process", in such a way that full disclosure starts from the player with the highest quality and goes down to players with lower quality (Grossman, 1981; Milgrom, 1981; Viscusi, 1978). However, these studies have not been able to fully explain the reality in markets where disclosure is incomplete, due to their strong assumptions (Dranove & Jin, 2010). For example, although these studies have assumed that customers are expected to be able to verify seller's statement ex post, it is not true in many markets where products sold are experience or credence goods, as it is often not easy for customers to evaluate the quality even after purchase. By focusing on the effect of risk, our paper more clearly suggests the incentive of low-quality sellers to disclose quality information even when customers cannot evaluate the quality. Therefore, it provides important understandings to the literature such as how quality disclosure can be independently initiated by low-quality sellers (i.e., how low-quality sellers disclose quality information even without unraveling effect), and under what circumstances information disclosure can be complete or incomplete considering the risk attitude of customers.

We believe our paper can also provide implications to the literature on cheap-talk communication between customers and sellers, where, unlike signaling, the disclosure of information is costless (Crawford & Sobel, 1982). Those papers on cheap talk have been exploring how sellers and buyers credibly communicate with each other when information disclosure is payoff-irrelevant, there is no credibility cost, and their incentives do not align (Aumann & Hart, 2003; Gardete, 2013; Li, 2005; Yi Zhu & Dukes, 2015). More specifically, these studies have suggested various tools that match the incentives of sellers and customers, thereby making the cheap-talk communication credible. For example, Li (2005) has found that a properly termed revenue-sharing contract removes the incompatibility of incentive and makes credible information transmission possible in a technology distribution channel, and Gardete (2013) has shown that advertising can align incentives of sellers and customers in a vertically differentiated market and thus deliver truthful messages to customers. In our model the information transmission also occurs via cheap-talk messages, and the incentives of sellers and buyers are misaligned as sellers want to hide weaknesses of their products while buyers would like to have that information.[5] It is the consideration of risk and risk attitudes of customers that play the role of matching buyers' and sellers' incentives and making credible information disclosure mutually beneficial. Therefore, our paper provides better understanding on the nature of cheap-talk communication by suggesting that consideration of risk can universally be applied to most cheap-talk cases as a tool for aligning the incentives of both parties.

Perhaps the most relevant findings can be found from several recent studies in marketing and economics that have examined the competitive structure and/or the heterogeneity of customers in order to understand the motivation of information disclosure. Board (2009) has shown that low-quality types may or may not disclose their types depending on whether the loss from lower perceived value is less than the gain from decreased competition with high-quality sellers, based on the assumption that consumers' tastes for quality vary and are uniformly distributed. Therefore, according to this finding, low-quality sellers might voluntarily disclose their types since they may gain from differentiating their products from high-type sellers and attracting those customers less sensitive with quality, even though their products are perceived as having low values. Guo and Zhao (2009) have also described the mechanism where duopoly sellers consider voluntarily disclosing quality information even when the quality is not the highest, in order to achieve differentiation and avoid direct competition. Gardete (2013) has shown that if customers differ in their marginal valuations for quality then different firms have different preferences over those consumer segments, suggesting that low-quality firms may want to reveal its types to attract those customers with low marginal valuations for quality. Kim (2012) has shown that in presence of search and matching frictions low quality sellers can reveal their type to attract more buyers and intensify comptition among them. Although demarketing is a broader type of marketing strategy than disclosure of low quality, Gerstner, Hess, and Chu (1993) have characterized a similar market phenomenon by explaining how demarketing can increase sales when customers are differentiated in their disutility to demarketing, as those with high tolerance prefer to choose demarketed products despite their low quality. On an empirical side, Tadelis and Zettelmeyer (2015) have performed a large scale field experiment in wholesale automobile auctions and proved that disclosure of quality information facilitates matching buyers with different preferences to quality to appropriate markets, thereby showing that even the disclosure of negative information can increase the revenue of sellers.

Although our paper has used similar approaches to these studies by focusing on the competitive structure of the market and buyers' heterogeneity, we believe that it provides a novel perspective to the literature by suggesting the effect of risk on consumer choices and the heterogeneous risk attitudes of customers as key factors affecting the impact of information disclosure. More specifically, our paper adds the following implications to existing literature. First, the discussion of the efficiency gain of information disclosure is more explicit in our model, compared to previous studies. The above mentioned literature has suggested that information disclosure might achieve efficiency gains since better information leads to a better sorting between consumers and products (Dranove & Jin, 2010). However, if customers are not sufficiently differentiated in their valuations for quality,

[5] Again, this is the main tension related to information disclosure in markets (Dranove & Jin, 2010)

those efficiency gains might be doubtful. Guo & Zhao (2009) have also pointed out that information disclosure by itself does not result in any efficiency gain in their model. On the other hand, by specifically considering the disutility from risk, our paper suggests that information disclosure not only helps matching customers with different tastes for quality, but also increases customer's utility through reducing risk from purchase, even when the information itself is negative. We also believe that the distribution of risk attitude among customers is more evident than customers' preferences towards quality, making our finding more applicable to real market situations.[6] Second, our study provides important strategic implications for marketing managers through explaining the way how disclosing low quality can attract customers, and thus help sellers. More specifically, our finding explains that revealing low quality might or might not increase sales depending on how much risk is reduced with the disclosure. Therefore, low-quality sellers might be able to attract more customers when they can more effectively reduce risk through various risk intermediaries if available, just as Ghose & Ipeirotis (2011) have found how negative product reviews can increase product sales when presented with informative and detailed text. Although the studies mentioned above have demonstrated possible incentives for revealing low quality, they have rarely suggested specific marketing strategy implications. On the other hand, this paper may propose meaningful marketing communication strategies to most sellers, as no product is perfect in customers' eyes and sellers always have to manage negative information about their products.

3 Monopoly

Consider the model with a seller that sells a product with exogenously given quality that is unobserved by buyers. The quality is distributed with a cdf $F(v)$ on an interval $[v_L, v_H]$ and the distribution can be either discrete, or continuous with a positive density on $[v_L, v_H]$. The marginal cost of the product depends on its quality. For a product of quality v we use c_v to denote its marginal cost. Unless explicitly stated otherwise, we assume that c_v and $v - c_v$ are strictly increasing functions of v. If the seller of type v serves share s_v of buyers at price p_v then his expected profit is

$$U_s = (p_v - c_v)s_v.$$

There is a continuum of buyers that we normalize to 1. Buyers' utility is determined by the price, p, at which they purchase the product, as well as their beliefs about product's quality, μ. Buyers are not risk-neutral and dislike the uncertainty about product's quality. We model buyer's uncertainty-aversion using two alternative frameworks. The first framework is a framework with loss-averse buyers whose reference point is endogenously determined by buyer's expectations (Shalev, 2000; and Köszegi and Rabin, 2006), which in our case is $E_\mu v$. If, given beliefs μ and price

[6] In other words, it might make more sense to reveal low quality in order to reduce risk and attract risk-averse customers than to disclose product defects in order to attract those customers who care less about quality, considering the vertically differentiated nature of quality preferences.

p, the buyer purchases the product of quality v then his utility is

$$u_b(v, p, \mu) = \begin{cases} v - p & \text{if } v \geq E_\mu v \\ (b+1)v - p & \text{if } v < E_\mu v \end{cases} \tag{1}$$

Here $b > 0$ measures the degree of loss-aversion. $E_\mu v$ is a reference point that determines whether the outcome is viewed as a gain or a loss. If the purchased product has quality below than what buyer had expected then the buyer experiences the loss. When $b = 0$ the buyer's utility has the slope of 1 in both the gains and losses domains. When $b > 0$ the buyer's utility is steeper in the loss domain, capturing the fact that buyers are more sensitive to losses. Buyers differ in their degree of loss-aversion. We assume that b is distributed with a positive differentiable log-concave density $\phi(b)$ and support $[0, B]$, where B can be infinity. Taking expectations of (1) over v we get

$$U_b^{LA}(p, \mu) = E_\mu v - p + b \cdot E_\mu[v - E_\mu v | v \leq E_\mu v] = E_\mu v - p - b \cdot ELoss_\mu, \tag{2}$$

where $ELoss_\mu = -b \cdot E_\mu[v - E_\mu v | v \leq E_\mu v] > 0$ is the buyer's expected loss and we define it so that it is positive. In our setting the loss-aversion framework with endogenous reference point is closely related to the perceived risk framework developed in the marketing literature (Dowling, 1986; Srinivasan and Ratchford, 1991), where perceived risk is defined as the probability of loss times the size of the loss from a purchase. It is also related to the disappointment-aversion framework by Gul (1991), which was one of the earliest papers to provide a theory of reference-point determination.

The second framework is the expected utility framework with risk-averse buyers. Buyers have concave Bernoulli utility function, $u(\cdot)$, with constant absolute risk-aversion. Buyers' utility from purchasing a product with uncertain quality v at price p is

$$U_b^{EU}(p, \mu) = E_\mu u(v - p). \tag{3}$$

Given the CARA assumption we do not need to specify the initial wealth. Buyers differ in the degree of absolute risk-aversion, γ. With a slight abuse of notations, we will use $\Phi(\gamma)$ to denote the distribution function of buyers' risk-aversion. Density $\phi(\gamma)$ is assumed to be positive, differentiable and log-concave.

Both loss-aversion and expected-utility frameworks capture the idea that buyers receive disutility when the product's quality is uncertain. The loss-aversion framework has a simpler functional form making it more tractable. However, just like disappointment-aversion or variance-aversion frameworks, the loss-aversion preferences can violate state dominance. For example, when b is high the buyer might prefer a product with known low quality over the product whose quality can be either low or high. It violates states dominance as the latter option will result in either the same (low) quality as the former option, or in the better (high) quality. Yet, buyers with high b will prefer the former. Our results, however, are not based on the violation of state-dominance as state-dominance holds in the expected-utility framework.

The timing of the model is as follows. First, the seller learns his type, v. Second, the seller sends a publicly observable cheap-talk message $m \in M$. Third, buyers observe the message and form posterior beliefs $\mu(m)$ about the quality distribution, which determines the demand for seller's product. Fourth, the seller chooses the price p and, finally, buyers decide whether to purchase the product or not.

Definition 1 *An equilibrium is a quadruple* $(m(v), p(m, v), \mu(m), s(\mu, p))$ *where* $m(v)$ *is the seller's messaging strategy,* $p(m, v)$ *is the seller's pricing strategy,* $\mu(m)$ *is the buyers' beliefs about the quality distribution, and* $s(\mu, p)$ *is the share of buyers who purchase the product such that the following conditions hold:*

a) given m *and buyer's demand* $s(\mu(m), p)$*, the seller of type* v *chooses price,* $p(m, v)$*, that solves his profit-maximization problem:*

$$\max_p (p - c_v)s(\mu(m), p);$$

b) the seller of type v *sends message* m *that maximizes his profit given buyers' beliefs and purchasing decisions*

$$\max_{m \in \mathcal{M}} (p(m, v) - c_v)s(\mu, p);$$

c) buyers purchasing decision is optimal, that is

$$s(\mu, p) = \Phi(\{b | U(p, \mu) \geq 0\});$$

d) if message m *is sent with positive probability buyers beliefs* $\mu(m)$ *are derived from* $m(v)$ *by Bayes' rule.*

Couple of remarks are due. First, we assume that prices are determined after the message, and not jointly. This assumption is appropriate if it takes more time to adjust the information disclosed, e.g. seller's advertisement strategy than it is to adjust prices. It is not essential for the case of a monopolistic seller. We do it impose it, however, for the sake of the similarity with the duopoly case where this assumption will make a difference as it will simplify the analysis of the price-setting decisions.

Second, messages are cheap-talk and we assume that prices cannot be used to signal the product's quality. One can see it in the condition d) of the definition whereby buyers' beliefs are determined solely by the cheap-talk message. Since prices do not change buyers' beliefs and sellers have different costs it means that different types of sellers will charge different prices that, technically, could be used to determine sellers' quality. In our setup, however, we assume away buyers' ability to infer quality from price. Our model, therefore, is more appropriate to the setting where buyers are inexperienced either because they are infrequent purchases of the product or where the products' prices are very volatile making it harder to use prices as signals. Later in the paper, we discuss what happens in our setup if prices are used as signals, but in the main framework we assume it away.

We impose this assumption for several reasons. First, theoretical literature on prices is well-established and has shown that under certain conditions, such as repeated purchases (Milgrom and Roberts, 1986) or in the presence of informed consumers, prices can allow high-quality sellers to credibly signal their high quality (Bagwell and Riordan, 1991). Given the focus of the paper on incentives of low-quality sellers and given the theoretically the role of prices as signals is well-understood we assume this possibility away. Second, having prices signaling the quality requires some level of buyer's sophistication and knowledgeability that can be unrealistic. They should be able to know the underlying cost and demand structure of the seller and its competitors and be sophisticated enough to use it to deduce the product's quality. This is different from the setup where only message, even if it is cheap talk, is used to provide information of quality. Messages such as "this oil is extra virgin", "this baseball card is grade 7" or "this car has a transmission problem" even when they are not supported by some verifiable information are easier to interpret even by unsophisticated buyers. Third, empirical literature on ability of prices to signal quality produces mixed results. Gerstner (1985) shows that quality/price relationship are weak and "*higher prices appear to be poor signals of higher quality*" (p. 214). Caves and Greene (1996) show that "*evidence consistent with price as a quality signal is confined to frequent but unimportant purchases*" (p. 29). Finally, Jin and Kato (2006) ran a field experiment with baseball cards to study the link between price, quality and seller claims. They found that when buyers pay higher prices they do not receive better quality. They also find that high-claim sellers target less-experienced buyers who, naturally, are less likely to be sophisticated enough to infer cards' quality from the price.

We begin now analysis of our framework. First, as the benchmark we look at the case of risk-neutral buyers. As Proposition 1 shows there is no equilibrium where any relevant information is transmitted, meaning that there is no equilibrium where seller's message changes buyer's valuation of the seller's product. This proposition is trivial so no formal proof is given. Informally, they reason why one cannot have an equilibrium with two on-equilibria messages which would lead to different expected qualities is that then all types would prefer the message with higher expected quality.

Proposition 1 *When all buyers are risk-neutral, for any on-equilibrium message m the average quality of type sending message m is Ev, $p(m) = Ev$ for all sellers, and all sellers serve the whole market.*

Next, we consider the framework of uncertainty-averse buyers. That affects sellers' incentives and ability to disclose information about its quality. In what follows, we say that m_s is a separating message if $Pr(v = v_s|m) = 1$ for some type v_s. Similarly, we say that m_p is a pooling message if there is more than one type that sends m_p. It might be possible for a given type to send both a pooling and separating messages with positive probability, which would require for this type to be indifferent between the two messages. As long as type v_s sends a separating message with positive probability we will say that v_s separates.

If seller with quality v_s sends a separating message m_s then buyers longer face no uncertainty about the seller's quality. All buyers value the product as v_s. Then the seller will set price $p = v_s$, all buyers will purchase the product and the seller will earn profit of $v_s - c_{v_s}$. Now, consider the case when some type v_p sends pooling message m_p. Let $\mu(m_p)$ be the quality distribution conditional on m_p. In the case of loss-averse buyers the demand function is given by the loss-aversion of a consumer who is indifferent between purchasing and not:

$$E_\mu v - p - b^0 ELoss_\mu = 0.$$

All buyers with $b < b^0$ will purchase the product and therefore the demand is equal to $\Phi\left(\frac{E_\mu v - p}{ELoss_\mu}\right)$. The optimal price for type v is then determined by the standard profit-maximization problem $\max_p \Phi\left(\frac{E_\mu v - p}{ELoss_\mu}\right)(p - c_v)$. One can similarly determine the demand function for the case of risk-averse buyers.

From the profit function above follows that sellers benefit if buyers have higher beliefs about the expected quality and lower beliefs about expected loss. This has two effects on sellers' incentives and ability to separate. First, for all types there is a benefit from separation as it increase buyers' willingness-to-pay for the product by removing the expected loss. Second, for sellers with higher quality it also means that they can charge a high price. The first effect is beneficial for all sellers. The second effect is more beneficial for sellers with higher quality. As the next Proposition shows, the second effect makes it impossible for high quality sellers — where by high-quality we mean with the quality above average — to separate. The benefits of separation are too high so that low-quality sellers — those with the quality below average — would strictly prefer to mimic the high-quality seller. For the low-quality sellers, however, the first effect is beneficial but the second effect can be detrimental. The former means that there are potential benefits from separation, and the latter means that benefits are not too high so that other sellers might choose not to mimic the separating type. Thus, in the case of the monopolistic seller and uncertainty-averse buyers only sellers with negative information, those with quality below average, can separate in equilibrium.[7] Later, we will provide examples that equilibria where sellers with low-quality separate exist.

Proposition 2 *In equilibrium at most one type separates. There is no equilibrium where type $v > Ev$ separate.*

The proposition below holds for both loss- and risk-averse buyers. That only one type can separate is trivial. A seller that separates will optimally set the price equal to its quality and will serve the whole market. If two types separate the one with lower quality will have incentives to mimic the one with higher quality. That only sellers with sub-average quality can separate comes from the following argument. If the separating type v_0 is the lowest type, then the proposition is trivial. If it is not, look at types with quality below v_0. Since these types choose not to mimic v_0 it means that they choose message that results in expected quality being above v_0. That is they pool with some higher quality types. All types that are not pooled with types in $[v_L, v_0)$ also send

[7] In the earlier version of the paper it was assumed that for purely exogenous reasons high-quality sellers cannot separate, and are exogenously restricted to sending one message. In that setup, any information transmission, including separation, was solely due to incentives of low-quality sellers. In the current version, all sellers are symmetric in what messages they can send. The inability of high-quality types to separate in this setting is now endogenous. Nonetheless, the case still can be made that the separation comes from incentives of low-quality sellers. The reason high-quality sellers do not separate is not that they do not have incentives to separate, but they can not to do so in equilibrium.

message that results in expected quality being above v_0, since they pool with types from $(v_0, v_H]$. Thus all types, but v_0 send on-equilibrium messages that result in expected quality being above v_0, which can only happen if $v_0 < Ev$.

As one would expect higher degree of loss- or risk-aversion is required in order for sellers with lower quality to be able to separate. Consider, for example, the seller of the lowest quality and the case of loss-averse buyers. If the seller chooses to separate he gets profit of $v_L - c_L$. It should be higher than profit from pooling with other sellers. Let m be a non-separating on-equilibrium message that leads to beliefs μ. Clearly, $E_\mu v > v_L$. The lower bound on the profit that seller v_L can earn when sending m is $E_\mu v - B \cdot ELoss_\mu - c_L$. This is because for given $(E_\mu v, ELoss_\mu)$ if a seller charges price $p = E_\mu v - B \cdot ELoss_\mu$ then it will serve the whole market. Since p is not necessarily optimal price given μ then $E_\mu v - B \cdot ELoss_\mu - c_L$ is a lower bound on the profit that v_L can earn given m. Since v_L prefers to separate it has to be the case that $E_\mu v - B \cdot ELoss_\mu - c_L \leq v_L - c_L$. And since $E_\mu v > v_L$ then it must be the case that B is sufficiently large.

The next Proposition describes properties of equilibrium strategies of the game. All types with low quality, and low cost, will set price low enough to serve the whole market. The price has to be the same for all these types. One cannot have equilibrium with two types serving the whole market at two different prices since the type charging lower price would have a profitable deviation. As for higher types, it follow from Proposition 3 that the equilibrium strategy has an interval structure: $\bar{v} = v_1 \leq \cdots \leq v_{N+1} = v_H$ where all sellers in interval $[v_i, v_{i+1}]$ send the same message m_i.

Proposition 3 *Assume that buyers are loss-averse with loss-aversion distributed on interval $[0, B]$. Without loss of generality assume that no two messages m and m' result in beliefs with identical Ev and $ELoss$. Let $\bar{v}$ be the highest quality type that serves the whole market, if it exists, and let $\mathcal{M}$ be the set of equilibrium messages sent by types with $v < \bar{v}$. Then*

i) if there are two different types $\bar{v} \leq v_1 < v_2$ that weakly prefer message m over any other message then any $v \in (v_1, v_2)$ will strictly prefer m over any other message;

ii) all types $v \leq \bar{v}$ charge the same price, serve the whole market and are indifferent between any message from $\mathcal{M}$.

We use Proposition 3 to give examples of equilibria where sellers with negative information separate. Consider a setting with three quality levels where $v_L = 0 < v_M < v_H$ and $c_i = \alpha v_i$. Assume that each quality level has probability of $1/3$ and that $Ev > v_M$. Buyers are loss-averse with $b \sim U[0, B]$. It is easy to determine parameter values when the equilibrium with v_M's separation exists. We look at the case of two on-equilibria messages: a pooling message m_p and a separating message m_s. When v_M separates it sets the price $p_M = v_M$ and serves the whole market earning profit $v_M - c_M$. By Proposition 3 type v_L must serve the whole market and charges the price p_M. Also by Proposition 3, both v_L and v_M must be indifferent between pooling and separating. This implies that in equilibrium v_M can mix between m_p and m_s. If v_M mixes then $\mu(m_p) = (\sigma, \tau, \sigma)$

where $\sigma = Pr(v = v_L|m_p) = (v = v_H|m_p)$ is between 1/3 and 1/2. Then the expected loss is

$$\begin{aligned} ELoss_\mu &= -[\tau(v_M - \tau v_M - \sigma v_H) + \sigma(v_L - \tau v_M - \sigma v_H)] \\ &= -\sigma(v_M\tau - v_H(\tau + \sigma)) = \sigma[(v_H - v_M) - \sigma(v_H - 2v_M)], \end{aligned}$$

and the expected quality is $E_\mu v = v_M + \sigma(v_H - 2v_M)$. Given m_p the demand faced by a seller is

$$s(\mu, p) = \frac{1}{B}\frac{E_\mu v - p}{ELoss_\mu} = \frac{1}{B}\frac{v_M + \sigma(v_H - 2v_M)}{\sigma[(v_H - v_M) - \sigma(v_H - 2v_M)]}. \tag{4}$$

Conditional on m_p, the optimal price to serve the whole market is $p_w = E_\mu v - B \cdot ELoss_\mu$ and it must be equal to v_M. Otherwise v_L and v_M would not be indifferent between pooling and separating.

Given m_p charging price $p = v_M$ must be optimal for both low- and medium-quality sellers. Since v_M has a higher cost it is the latter condition that is binding. It is immediate to derive the exact condition:

$$\frac{\sigma(v_H - 2v_M) + v_M - c_M}{2\sigma(v_H - 2v_M)} = \frac{\sigma(v_H - 2v_M) + v_M - \alpha v_M}{2\sigma(v_H - 2v_M)} \geq 1. \tag{5}$$

From $E_\mu v - B \cdot ELoss_\mu = v_M$ we can solve for σ, which is equal to

$$\sigma = \frac{v_H - v_M}{v_H - 2v_M} - \frac{1}{B}$$

and plug it into (5) we get

$$\alpha v_M \leq (v_H - 2v_M)\frac{1 - B}{B}. \tag{6}$$

Thus, if parameters are such that $\sigma \in [1/3, 1/2]$ and (6) is satisfied an equilibrium where type v_M separates exists. We did not check incentives for type v_H but it is straightforward to see they that it is optimal for v_H will pool. If v_H sends m_P and sets $p = v_M$ it earns $v_M - c_H$, which is also its separating profit. This is a lower bound on v_H's profit from pooling since his optimal price can differ from $p = v_M$. Thus H weakly prefers m_p over m_s. The interpretation of (6) is straightforward. For example, B cannot be too large ($B \leq 1$) as having too loss-averse buyers would would make the separation too attractive. Higher v_H makes pooling more attractive thereby preventing type v_L from separating, which is why (6) is easier to satisfy when v_H is higher. Note, however, that since $B \leq 1$ and $\sigma \in [1/3, 1/2]$ we have that $v_H/v_M \leq 4$.

The example above can be easily adjusted to the case of risk-averse customers. The primary difference between risk-averse and loss-averse buyers is that with loss-averse buyers it is possible to have an equilibrium where the lowest type reveal its information while with risk-averse buyers such an equilibrium is impossible.

Indeed, assume that the lowest type can separate itself with message m_L, and then it charges price $p = v_L$ and earns profit $v_L - c_L$. Consider another on-equilibrium message that generate beliefs μ with $E_\mu v$. Given that the lower bound of support of μ is greater or equal than v_L and it has a positive measure of sellers with higher quality $E_\mu u(w + v - v_L) > u(w)$ for any concave

$u(\cdot)$. But then if type v_L were to send message m and charge price $v_L + \varepsilon$ it would still serve the whole market but would earn a higher profit. The difference between loss-aversion and risk-aversion cases is that risk-aversion respects state-dominance. Risk-averse will always prefer a product that quality can be either v_L or something better to a product with certain quality of v_L. Which is why it is never optimal for v_L to separate. For loss-averse buyers, on the other hand, if degree of loss-aversion is high enough then these buyers would prefer a product of certain quality v_L over a product that can be either v_L or better.

4 Duopoly

4.1 Equilibrium and Its Properties

Next we consider how competition affects sellers' incentives to reveal negative information. There are two sellers and each seller has one unit of a product. The quality of each product is exogenous and is unobserved by buyer. Just like in Board (2009) or Janssen and Teteryatnikova (2016) we assume that sellers know each other quality.[8] The timing of the model is follows. First, both sellers simultaneously send costless messages (m_i, m_j) that are publicly observed. Second, given the messages sellers simultaneously determine prices for their products (p_i, p_j). Finally, buyers observe the both messages and both prices and choose which product to purchase. Utility of buyers and sellers is the same as in the monopoly section. Sellers' objective function is their expected profit and buyers can be either loss- or risk-averse.

In the case of duopoly sending messages before setting prices implies that competitors can react with their pricing decisions to the type of information disclosed (Janssen and Teteryatnikova, 2016).

Definition 2 *An equilibrium is of the duopoly model is a set of messaging and pricing strategies, $m_1, m_2, p_i(m_i, m_j, v_i, v_j), p_j(m_i, m_j, v_i, v_j)$; buyers' beliefs $\mu_i(m_i), \mu_j(m_j)$; and a purchasing strategy of the consumers $s_1(\mu_1, \mu_2, p_1, p_2), s_2(\mu_1, \mu_2, p_1, p_2)$ such that*

a) seller i of type v_i sends message m_i that maximizes his profit given seller j's messaging and pricing strategies, buyers' beliefs and purchasing decisions

$$\max_{m \in \mathcal{M}} (p_i(m_i, m_j, v_i, v_j) - c_{v_i}) s_i(\mu_i, \mu_j, p_i, p_j);$$

b) given m_i, buyers' beliefs and purchasing strategies, and given seller j messaging and pricing strategies seller i of type v_i chooses the price that solves his profit-maximization problem:

$$\max_{p} (p - c_{v_i})) s_i(\mu_i, \mu_j, p_i, p_j);$$

c) buyers purchasing decision is optimal, that is

$$s_i(\mu_i, \mu_j, p_i, p_j) = \Phi(\{b | U(p_i, \mu_i) \geq 0 \text{ and } U_b(p_i, \mu_i) \geq U_b(p_j, \mu_j)\}),$$

[8] Earlier version of the paper assumed that the quality was pure private information and unobserved by the other seller. The assumption that the quality is observable greatly simplifies the analysis of the pricing stage subgame in a general setting. Pinto and Parreira (2015) provide some results on price competition in the Hotelling model with uncertainty about the other seller's type.

and $s_j(\mu_i, \mu_j, p_i, p_j) = 1 - s_i(\mu_i, \mu_j, p_i, p_j)$.

d) if message m_i *is sent with positive probability buyers beliefs about quality of seller* i, $\mu_i(m_i)$, *are derived from* $m_i(v)$ *by the Bayes' rule.*

As the definition of equilibrium reveals we made several simplifying assumptions. First, we assume away the possibility of comparative advertising, or at least, the possibility of effective comparative advertising. Buyers use seller's message only to update their beliefs about seller's type and not seller's competitor type. Allowing for comparative advertising would complicate the model without adding much to the main insight of the paper.[9] Second, as in the monopoly case we assume that prices are determined after the message, and not jointly. The assumption is reasonable as long as it is quicker to adjust pricing strategy so that sellers can react with their pricing decision to the type of information disclosed.[10] Finally, as in the monopoly case we assume away the possibility of using prices as a signaling device, as buyers do not use prices to update their beliefs about seller's quality.

As in the case of monopoly, we first analyze the setting when buyers are risk-neutral. We show that no information about expected quality can be transmitted in equilibrium. Intuitively, if there are two on-equilibria messages of seller i that result in different beliefs about expected quality then all types of seller i would prefer a message with the highest expected quality.

Proposition 4 *When buyers are risk-neutral then* $E_{\mu(m_i)} v_i = Ev$ *for any on-equilibrium message* m_i.

Just like in the case of a monopoly one can show that in any equilibrium at most one type can separate. The intuition, however, is somewhat different from the monopoly case. Similar result can be shown in the case of duopoly. In the proof we demonstrate that for any pricing subgame it is more profitable to pretend to be of a higher type, which means that at the messaging stage one cannot have two types separating. Arguably, the result is more important in the case of discreet quality distribution since it implies that separation occurs with a positive probability.

We analyze the case for loss- and risk-averse buyers as two separate propositions mostly because the proofs differ. The case of loss-averse buyers, being simpler, allows for an explicit solution of the pricing stage equilibrium and is, therefore, more illustrative. The case of risk-averse buyers does not permit an explicit solution so instead we rely on properties of sellers' best responses and the envelope theorem.

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[9] The literature on comparative advertising has mostly mixed findings (Grewal et al., 1997). With some authors (e.g. Swinyard, 1981) showing that comparative advertising is perceived as less credible, or that it does not generally differ from non-comparative advertising (Sujan and Dekleva, 1987). With our assumption we do not necessarily preclude any comparative advertising but rather comparative advertising that is informative about the competitor's quality.

[10] When sellers know each other types this assumption is rather innocuous. In pure strategy equilibria it does not matter when prices and messages are sent simultaneously or sequentially. It has some bite when an equilibrium is mixed, for example, when a given type mixes between two different messages. Then separating the messaging and pricing stages simplifies the analysis of the price setting decision in the same way as the assumption that sellers know each other types does.

Proposition 5 *In a symmetric equilibrium with two sellers and loss-averse buyers at most one type can separate.*

Proposition 6 *In a symmetric equilibrium with two sellers and risk-averse buyers at most one type can separate.*

4.2 Comparing Monopoly and Duopoly settings. Examples

The results established so far show similarity between the monopoly and duopoly cases. For example, in both the monopoly and duopoly settings it is impossible to have two types that separate in equilibrium. The reason is that the types that separate have the same uncertainty about their product's quality, which is none, but one of the separating types will have a higher quality and a lower type who chooses to separate would then find it optimal to mimic a higher separating type.

Nonetheless the two settings differ as competition creates a new incentive for sellers to separate. The incentive comes from the fact that now quality disclosure achieves two goals. First, as before, it removes buyers' uncertainty about the product. Second, it affects the intensity of competition with the other seller by creating a product differentiation. Consider, for example, an equilibrium where all types pool. In this equilibrium buyers' information about both products' quality is identical and they will purchase the cheapest product resulting in the intense Bertrand competition. This in contrast to the case when seller i sends a separating message while seller j sends a pooling message. The two products then are differentiated. Buyers with low degree of uncertainty-aversion are primarily concerned with the price and expected quality. Buyers with high degree of uncertainty-aversion dislike the uncertainty about j's quality and might choose to purchase product i, even if it has lower quality, or if it is more expensive. This softens the competition benefiting both sellers.

As we will show the fact that separation affects the intensity of competition allows two outcomes to become possible that were not possible under the monopoly setting. First, there are equilibria where the type with the highest quality can separate. Second, in the case of risk-averse buyers the lowest quality seller can separate. In both cases, the reason why such equilibria are possible is that pooling with higher-quality sellers can be less attractive if it is more likely to result with an intensive competition with the other seller. But then if pooling with high quality sellers is less attractive it becomes possible for high-type to separate or, in the risk-averse case, for the lowest quality type to choose separation.

To demonstrate a possibility of such outcomes it will suffice to consider a very simple setup where sellers have two quality types v_L and v_H, with $v_L < v_H$, and costs of both types are zero. We will assume that the probability of high-quality is q.

4.2.1 Expected Utility Framework. Separation of low-quality type

Consider the following candidate for an equilibrium. High-quality seller sends message H with probability 1, and low-quality seller mixes messages L and H with probabilities λ and $1 - \lambda$. Message L is the separating message as in equilibrium only type v_L sends this message. In what follows whenever we refer to message we use L or H. When referring to quality we will be using low- and high-quality or v_L and v_H interchangeably. Buyers are risk-averse with the CARA utility function $u(x) = 1 - e^{-\gamma x}$, where $\gamma \sim \Phi(\gamma)$ with support $[0, \Gamma]$.

We solve for equilibrium using backward induction. First, we look at the purchasing stage. If both sellers send the same message (L, L) or (H, H) then from buyers' point of view both sellers are identical. They will purchase the product with the lowest price. Consider now the subgame after (L, H). The L-product has a certain quality of v_L. The H-product can be of low quality with probability $q_{LH} = Pr(v = v_L|H)$ or of high quality with probability $1 - q_{LH}$. Given the assumed messaging strategies,

$$q_{LH} = \frac{(1-\lambda)(1-q)}{(1-\lambda)(1-q)+q}.$$

A buyer is indifferent between L and H products if

$$u(v_L - p_L) = q_{LH} u(v_L - p_H) + (1 - q_{LH}) u(v_H - p_H),$$

or equivalently

$$e^{-\gamma^0(p_H - p_L)} - q_{LH} - (1 - q_{LH}) e^{-\gamma^0(v_H - v_L)} = 0. \tag{7}$$

where γ^0 is the risk-aversion degree of an indifferent buyer. Given the CARA utility function the buyers' initial wealth does not affect the indifference condition so we do not specify it.

Proposition 7 *The indifference condition*

$$e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH}) e^{-\gamma^0(v_H - v_L)},$$

has at most one solution $\gamma^0 > 0$. When the solution exists, all types with $\gamma > \gamma^0$ prefer an L-product while all types with $\gamma < \gamma^0$ prefer H-product.

Now we move to the analysis of the pricing subgame. If during the messaging stage both sellers sent the same message (L, L) or (H, H) then, as we established earlier, buyers will purchase the cheapest product and, therefore, the pricing equilibrium is for both sellers to charge $p_L = p_H = 0$. Consider now the (L, H)-subgame. The optimization problem of the L-seller is $\max_{p_L}(1 - \Phi(\gamma^0(p_L, p_H)) \cdot p_L$, and the optimization problem of the H-seller is $\max_{p_H} \Phi(\gamma^0(p_L, p_H)) \cdot p_H$. The corresponding first-order conditions are

$$-\phi(\gamma^0) \frac{\partial \gamma^0(p_L, p_H)}{\partial p_L} p_L + (1 - \Phi(\gamma^0)) = 0, \tag{8}$$

and

$$\phi(\gamma^0) \frac{\partial \gamma^0(p_L, p_H)}{\partial p_H} p_H + \Phi(\gamma^0) = 0. \tag{9}$$

It follows from (7) that

$$\frac{\partial \gamma^0}{\partial p_L} = -\frac{\partial \gamma^0}{\partial p_H} = -\frac{\gamma^0 e^{-\gamma^0(p_H - p_L)}}{-(p_H - p_L) e^{-\gamma^0(p_H - p_L)} + (1 - q_{LH})(v_H - v_L) e^{-\gamma^0(v_H - v_L)}}.$$

Finally, at the messaging stage low-quality seller i should be indifferent between L and H when seller j plays the equilibrium strategy. This condition is

$$\lambda(1-q)\pi_{LL} + (1-\lambda(1-q))\pi_{LH} = \lambda(1-q)\pi_{HL} + (1-\lambda(1-q))\pi_{HH}. \tag{10}$$

Here the LHS is the expected profit of the low-quality seller from sending message L and the RHS from sending message H; $\lambda(1-q)$ is the probability that seller j sends message L, and $(1-\lambda(1-q))$ is the probability that seller j sends message H. As we established earlier $\pi_{LL} = \pi_{HH} = 0$. Notice that since both types have the same cost, the high-quality seller is also indifferent between L and H and, therefore, message H is optimal. Combining equations (7), (8), (9) and (10) we get that an equilibrium is determined by the following system of equations:

$$\begin{cases} e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)}. \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_L + (1-\Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \\ (1-q_L)(1-\Phi(\gamma^0))p_L = q_L\Phi(\gamma^0)p_H. \end{cases} \tag{11}$$

As the next Proposition shows whether the solution to the system above exists depends on the distribution of γ.

Proposition 8 *Assume that there are two quality types, and buyers are risk-averse with CARA utility function. Then the equilibrium where the lowest-quality type separates does not exist if*

i) $\Phi(\gamma)$ *is a uniform distribution; or*

ii) $\Phi(\gamma)$ *is a convex function.*

The equilibrium where the lowest-quality type separates exists if

iii) $\Phi(\gamma)$ *has infinite support; alternatively*

iv) for any concave $\Phi(\gamma)$ *with finite support there exists* $\alpha^0 > 0$ *such that for any* $\alpha \in (0, \alpha^0)$*, if risk-aversion is distributed with cdf* $\Phi(\alpha\gamma)$ *then an equilibrium with negative information exists.*

The intuition is as follows. In terms of quality, the H-product is superior to the L-product since the L-product is guaranteed to be of low quality, while the H-product can be of either low- or hiqh-quality. Regardless of γ, all risk-averse buyers prefer the H-product. However, for those who are more risk-averse the difference in expected utilities between purchasing the H- and the L-products is smaller. Thus, the only way the L-seller can get a positive share of the market is by competing with the H-seller for risk-averse buyers with high γ, (see also Proposition 7). The H-willingness to compete for buyers with high risk-aversion depends on two factors: how risk-averse are the buyers with high risk-aversion, and how large is their share. When Γ is low then the cost of competing these buyers is low, since buyers are not too concerned about quality uncertainty. A small increase in p_H can result in large gains in the market share. In the extreme case of $\Gamma = 0$, for example, the H-seller will always outprice the L-seller by charging price $p_H = E_H v - v_L$, and serving the whole market. Assuming that Γ is sufficiently high, however, is not enough. The share of buyers who are too risk-averse cannot be too high. If the share of extremely risk-averse buyers is low then it is more profitable for the H-seller not to compete for these customers and, instead, charge a higher price and serve a larger group of less risk-averse customers. Then, in equilibrium the H- and L-sellers split the market. If, however, the share of extremely risk-averse customers is too high then the H-seller will find it optimal to compete for them and he will outprice the L-seller.

Proposition 8 captures the notion of having "*positive but sufficiently low share of sufficiently risk-averse buyers*" using convexity and support of $\Phi(\gamma)$. First, for any $\Phi(\gamma)$ with infinite support the equilibrium with the lowest-type separation exists. There will always exist $\tilde{\gamma}$ high enough so that the share of buyers with $\gamma > \tilde{\gamma}$ is too low, making H-sellers less interested in competing for those buyers. Second, for uniform or convex cdfs, there are too many buyers with high degree of risk-aversion preventing the equilibrium existence. No matter how high Γ, there are too many buyers with sufficiently high degree of risk-aversion and the H-seller will always find it optimal to compete for them and outprice the L-seller. As for concave distributions, concavity alone is not enough to guarantee existence. One also need to have sufficiently risk-averse buyers in the support. Otherwise, when the support's upper-bound is too low there might not be enough risk-averse buyers to make the disclosure worthwhile. One way to achieve it is by stretching $\Phi(\gamma)$ to a larger support, and Proposition 8 offers one way how it can be achieved. Notably, as the next example shows, one does not need unrealistic levels of risk-aversion for the equilibrium existence.

Example 1 *Let the degree of risk-aversion γ be distributed with $\Phi(\gamma) = \sqrt{\gamma}$ on $[0, 1]$. The highest degree of risk-aversion among buyers, therefore, is 1. Let $v_H - v_L = 3$ and $q \approx 0.484$.*[11] *One can verify that the following is equilibrium: $\gamma^0 \approx 0.713$, $p_L \approx 0.184$, $p_H \approx 1$, $\lambda \approx 0.063$ and $q_{LH} = 1/2$.*

In this example, the probability of buying from a low-quality seller is slightly above 1/2, $1 - q \approx 0.52$. The low-quality sellers reveals the negative information with probability 6.3%. If one seller announces L and another announces H then both prices are above the marginal cost and both sellers making positive profit. The indifferent buyer is located at $\gamma^0 \approx 0.713$ so that the L-announcer has $1 - \sqrt{\gamma^0} \approx 15.5\%$ share of the market and the H-announcer has 84.4% of the market. In the (L, H) subgame sellers profits are $\pi_H \approx 0.84$ and $\pi_L \approx 0.03$.

It is worth highlighting that what makes it is competitive that environment that makes it possible for seller v_L. In the monopoly case with risk-averse buyers, if v_L does not separate and pools with v_H it is guaranteed to get profit higher than v_L which its separating profit. Thus pooling is preferable. In the duopoly case, if for both sellers v_L pools with v_H then it earns zero profit as the sellers engage in the Bertrand competition. By sending message L, type v_L creates product differentiation allowing it to earn positive profit.

[11] Numerically, it is simpler to start with q_{LH} instead of q. The equilibrium in this example was calculated based on $q_{LH} = 1/2$. Value of q and λ were then calculated from (γ^0, p_L, p_H) and q_{LH} using the fourth equation of (11).

4.2.2 High-quality seller separating.

Since quality disclosure can soften the competition it allows for equilibrium where high quality sellers can separate. If the high-quality segment of the market is very competitive, then there is a cost of mimicking high type, which is the competitor is very likely to be a high-type as well. If buyers believe that both sellers are of high type it results in Bertrand competition and zero profit. This can allow high-quality type to separate with positive probability as intense competition will discourage low-quality types from mimicking it.

As in the subsection above, we use a setting where sellers have two quality types and zero marginal costs. For simplicity, however, we assume that buyers are loss-averse. Consider the following messaging strategies: a high-quality seller randomizes between sending messages L and H, while low-quality seller sends message L with probability 1. Let λ be the probability of high quality seller sending message L, and $q_{LL} = Pr(v = v_L|L) = \dfrac{1-q}{1-q+q\lambda}$.

As before, buyers will purchase the product with the lowest price after (L, L) and (H, H) messages, so that both sellers charge prices equal to the marginal cost and earn zero profit. Consider the (L, H) case. The loss-aversion of the indifferent buyer is given by

$$v_H - p_H = q_{LL}v_L + (1-q_{LL})v_H - p_L + b^0 q_{LL}(v_L - q_{LL}v_L - (1-q_{LL})v_H).$$

Buyers with $b > b^0$ will purchase from seller H and buyers with $b < b^0$ will purchase from seller L so that the profit of the H-seller is $(1-\Phi(b^0))p_H$, and of the L-seller is $\Phi(b^0)p_L$. Combining the buyers' indifference condition and the FOCs for L and H-sellers we get

$$\begin{cases} b^0 = \dfrac{p_H - p_L}{q_{LL}(1-q_{LL})\Delta v} - \dfrac{1}{1-q_{LL}} \\ \phi(b^0)\dfrac{\partial b^0(p_L, p_H)}{\partial p_L} p_H + (1-\Phi(b^0)) = 0 \\ \phi(b^0)\dfrac{\partial b^0(p_L, p_H)}{\partial p_L} p_L + \Phi(b^0) = 0 \end{cases} \tag{12}$$

We do not write the indifference condition because one use the equivalent of Lemma 2 to show that if solution to (12) exists then the equilibrium exists.

By subtracting the second equation from the third equation and taking into account the definition of b^0, and its derivative we get

$$\frac{1-\Phi(b^0)}{\phi(b^0)} - \frac{\Phi(b^0)}{\phi(b^0)} - b^0 = \frac{1}{1-q_{LL}}.$$

For a given q_{LL} the solution to (12) exists if and only if

$$\max_b \left\{ \frac{1-\Phi(b)}{\phi(b)} - \frac{\Phi(b)}{\phi(b)} - b \right\} > \frac{1}{1-q_{LL}}.$$

For distributions with log-concave densities the expression inside the parenthesis is a decreasing function of b^0. By Theorem 1 of Bergstrom and Banoli (2006) if $\phi(b)$ is log-concave then so is $\Phi(b)$.Term $(1-\Phi(b))/\phi(b)$ is a decreasing function of b by Corollary 2 in Bergstrom and Banoli

(2006). Term $\Phi(b)/\phi(b)$ is an increasing function of b by definition of log-concavity, and its negative, therefore, is decreasing. Therefore, the maximum of the expression in the parenthesis is reached at $b = 0$. Thus, for a given q_{LL} the equilibrium exists if

$$\frac{1}{\phi(0)} > \frac{1}{1 - q_{LL}}.$$

Finally, since $q_{LL} \geq 1 - q$ the equilibrium where high type separates exists when

$$\frac{1}{\phi(0)} > \frac{1}{q}. \tag{13}$$

We can now interpret (13). Product H is vastly superior to product L: it has a higher *and* certain quality. For sellers to be willing to send message L, L-product should be able to compete with the H-product. For that, first of all, q cannot be too low. The only way L-seller can make profit when competing with the H-seller is if its product differentiated from product H which, in our model, requires the L-product to have quality uncertainty. Without quality uncertainty about the L-product, the H-seller would get the whole market. When q is low then q_{LL} is close to 1 which means that the L-product is almost certainly low-quality, thus a L and H sellers will not split the market and sellers would never choose message L. When q is sufficiently high, on the other hand, L is sufficiently differentiated from H and it is possible to make positive profit. Second, $\phi(0)$ should be sufficiently low. The intuition is similar to that for the equilibrium where v_L separated from the previous section. The only difference is that in the previous case the seller of the inferior product, L-seller, was competing for buyers with high-degree of risk-aversion, and now it competes for buyers with low degree of loss-aversion.[12] Therefore, by the same logic as before there cannot be too many buyers with low degree of loss-aversion, otherwise seller H would find it optimal to compete for them. When $\phi(0)$ is low, then it is not profitable for the H-seller to compete for risk-neutral buyers as there are too few of them. The H-seller instead prefers to charge a higher price for loss-averse buyers, thereby allowing the L-seller to get some share of the market. Finally, while condition (13) does not explicitly depend on B but it might depending on the distribution. For example, when $b \sim U[0, B]$, condition (13) becomes $B > 1/q$. Thus, for (13) to be satisfied and for the equilibrium to exist a necessary condition is that $B \geq 1$. At the same time when $\Phi(b) = (b/B)^2$ then condition (13) is satisfied for any B and any q as $\phi(0) = 0$.[13] Given the negligible amount of buyers with low degree of loss-aversion it is never optimal for the H-seller to serve the whole market.

Figure 1 depicts equilibrium parameters in equilibria where high-quality seller separating. It is plotted for $B = 3$, and $\Delta v = 3$. On the horizontal axes is q_{LL} which is probability of getting

[12] The fact that last subsection considered the case of risk-averse buyers and this subsection the case of loss-averse buyers is not essential. It is straightforward to verify that when buyers are loss-averse then the equilibrium where type v_L separates exists iff $B + \frac{1}{\phi(B)} > \frac{1}{1-q}$. In other words, in that case value of $\phi(B)$ had to B sufficiently small. What matters is not risk- or loss-aversion but whether the inferior product has certain quality — then the L's niche is buyers with high loss or risk-aversion — or if it has uncertain quality — then the L's niche is buyers with low loss- or risk-aversion.

[13] This is almost exact equivalent of Proposition 8 c) which says that if $\Phi(\gamma)$ has infinite support the equilibrium exists, as then $\phi(\Gamma) = \phi(\infty) = 0$.

low-quality product from the L-seller. We use q_{LL} and not q for the horizontal axes because as one can see from Figure 1, q has to be extremely close to 1 in order for an equilibrium to exists. One can also see that the price of the H-product is much higher than that of the L-product. This is not surprising given that the H-product is superior to the L-product. The reason message L is sent in an equilibrium is that very low λ, that is high-quality sellers is very likely to send H. Then the high profit in the (H, L) subgame comes at cost of very high likelihood of the (H, H) subgame and zero profit. For the L-seller, a low profit in the (H, L) subgame comes with benefit of having very low likelihood of being in the (L, L)-subgame and earning zero profit.

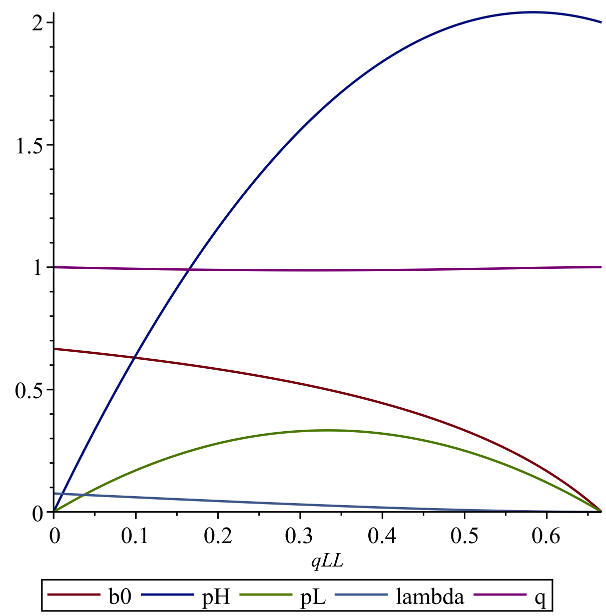

Figure 1: Equilibrium parameters when high-quality seller separates. We assume $b \sim U[0, 3]$ and $v_H - v_L = 3$. We use q_{LL} and not q for the horizontal axes because there is not much variation in q as it is has to be too close to 1.

5 Extension. Prices used as signals.

Throughout most of the paper, we assumed away possibility that prices can be used as signals, as only buyers use only cheap-talk messages to update their beliefs. Here we briefly discuss what happens when buyers use prices to update their beliefs. For simplicity we assume that buyers are loss-averse, and the seller is monopolist.

Assume that qualities are distributed on interval $[v_L, v_H]$. The marginal cost of type with quality v is c_v. The timing of the game is as follows. First, the seller learns his type. Second, the seller announces the price for his product. Third, buyers observe the price and form posterior beliefs $\mu(p)$ about the quality distribution conditional on p and decide whether to purchase the product or not. All other aspects of the model are as before.

Definition 3 *An equilibrium is a triple* $(p(v), \mu(p), s(\mu, p))$*, where* $p(v)$ *is the seller's pricing strategy,* $\mu(p)$ *is the buyers' beliefs about the quality distribution given* p*, and* $s(\mu, p)$ *is the share of buyers who purchase the product, such that the following conditions hold:*

a) the seller of type v *chooses price* p *that maximizes his profit given buyers' beliefs* $\mu(p)$ *and buyers' purchasing decision:*

$$\max_p (p - c_v) s(\mu(p), p);$$

b) buyers purchasing decision is optimal:

$$s(\mu, p) = \Phi(\{b | U(p, \mu) \geq 0\});$$

c) if a given price p *is chosen with positive probability buyers beliefs* $\mu(m)$ *are derived from* $p(v)$ *by Bayes' rule.*

The case when c_v is an increasing function of v was intensively covered in the literature and is well-understood. When the single-crossing condition satisfied and it is the informed party that makes an offer sellers can separate even in the case of risk-neutral buyers. Unlike the cheap-talk setting considered above neither uncertainty-aversion nor competition is necessary for separation. A simple example below illustrates this point.

Example 2 *Assume there are two types:* $\{L, H\}$ *with* $v_L = 1, c_L = 0$ *and* $v_H = 2, c_H = 1/2$*. The separating prices are* $p_L = 1$ *and* $p_H = 2$*. The corresponding sales are* $q_L = 1$ *and* $q_H = 1/2$*. This is (weakly) optimal for buyers as in both cases they are indifferent between purchasing or not. The corresponding profits are* $\pi_H = 3/4$ *and* $\pi_L = 1$*. If type* H *mimics type* L *its profit is strictly lower,* $1/2$*. If type* L *mimics type* H *its profit is the same, 1. The off-equilibrium beliefs that support this equilibrium are:* $\Pr(v_H|p) = 1$ *for any* $p \geq 2$ *and* $\Pr(v_H|p) = 0$ *for any* $p < 2$*.*

In order for loss-aversion to add to the set of possible outcome we need to consider the case when single-crossing condition is violated. We will assume the most extreme case that the cost is *decreasing* function of quality.[14] A standard argument then can be used to show that any equilibrium will have an interval structure whereby either a given type separates or, if two types pool and set the same price, then all types in between will strictly prefer to use the same price. The first part of Proposition is fairly standard and its proof does not depend on buyers' preferences and on whether the quality distribution is discreet or continuous. The only assumption that matters is the strict monotonicity of the cost function. The second part depends on the fact that the cost function is decreasing. Then in order for types to send different messages, types with the higher cost/lowest quality should charge a higher price and serve lower share of the market. Were it not

[14] While this is not a standard assumption in a theoretical literature, it is not unheard of in the empirical literature. A textbook (Besanko et al., 2009) example is Miller and Friesen (1986)'s study of consumer durable industries. Miller and Friesen (1986) showed that firms that appeared to have achieved cost advantage also scored highly on measures related to benefit superiority, such as product quality. See further discussion in Besanko (2009, p. 389) including empirical evidence and factors that allow firms to produce higher-quality products at lower costs, such as economy of scale or experience curve.

the case, types with lower costs would find it optimal to mimic them. The last remark is that for the purposes of this section it does not matter how many types can separate in equilibrium, so we do not investigate whether N has to be finite or not.

Proposition 9 *For any equilibrium:*

i) there exist $v_1, \dots, v_{N+1}$ such that $v_L = v_1 \leq v_2 \leq \dots \leq v_N \leq v_{N+1} = v_H$, where N can be infinite; all sellers with $v \in (v_i, v_{i+1})$ choose the same price, which we denote as p_i, with probability 1. If type distribution is continuous and $1 < i < N + 1$ then type $v = v_i$ is indifferent between between p_i and p_{i-1};

ii) $p_1 > \dots > p_N$. Let s_i be the equilibrium sales of sellers with $v \in [v_i, v_{i+1}]$. Then $s_1 < \dots < s_N$.

Next two Propositions show when buyers are loss-averse more information can be transmitted in the equilibrium then in the case of risk-neutral buyers.

Proposition 10 *In any equilibrium with risk-neutral buyers there can be at most two different on-equilibrium prices. An equilibrium with two on-equilibrium prices exist.*

Proposition 11 *Fix N. When buyers are loss-averse there exists an equilibrium with N on-equilibrium prices.*

6 Conclusion

Although conventional wisdom holds that negative information hurts sellers, many sellers still voluntarily share negative information and claim low quality in the markets where information asymmetry is high. This is somewhat counterintutive as sellers may get damaged by such a disclosure especially when customers cannot easily evaluate true quality in these markets. This paper has analyzed how honesty can be beneficial for low-quality sellers. The novel aspect of our model is the focus on incentives of low-quality sellers. To that end, standard tools that high-quality sellers can use to credibly reveal their quality — reputation, repeated purchases, warranties, using prices as signals — are not available. The only tool at sellers' disposal is cheap-talk messages, e.g. advertising, and this is only information used by buyers to update their beliefs. A standard result in this setup is that with risk-neutral buyers no information transmission is possible: there is no equilibrium where buyers change their valuation of the product based on a cheap-talk message.

We identify two factors that can allow low-quality sellers to prefer separation over pooling with high-quality sellers. The two factors are buyers' uncertainty-aversion and competition. In the case of the monopoly seller, with no competition, revealing one's quality removes uncertainty and, therefore, increases buyers' willingness-to-pay. The competition brings an additional effect as quality disclosure introduces product differentiation of competing sellers, thereby softening the competition. We show that in the monopoly setting only negative information — that the quality is below average — can be fully disclosed. Benefits of full disclosure would be too high for high-quality sellers making it impossible to prevent low-quality sellers from mimicking them. Competition changes incentives in that mimicking high-quality type is not necessarily optimal, if high-quality types face more intensive competition. This allows for two new outcomes that were impossible in the case of the monopolistic sellers: high-quality type can separates and, in the case of the risk-aversion buyers, lowest-quality can choose to separate. In both cases the reason it can happen in equilibrium is exactly because if it is less profitable to pretend to be high type.

We believe that the findings of this paper can be universally applied to numerous real market cases, since our assumption of costless cheap talk is actually the case in many markets where information asymmetry is high. Furthermore, the findings of this paper are expected to provide valuable insights to both the field and the academia on the behaviors of sellers in the markets under information asymmetry. For marketing managers, this paper can provide helpful strategic suggestions on how to deal with the information about the weaknesses of their products. This is an important issue for most marketers as no product or service is perfect in customers' eyes and sellers always have to deal with certain negative information. For policy makers, this paper suggests a new perspective on how to effectively deal with frauds in markets where information asymmetry is high. According to our results, the level of market fraud might be affected by several factors such as the ratio of high quality products and the level of risk-sensitivity of customers, and thus policy makers might have to measure and monitor those factors in order to cope with potential frauds. For academic researchers, this paper provides a theoretical explanation on the important market phenomena that have been somewhat neglected by the literature. More specifically, this paper suggests that we observe different levels of frauds across markets in different industries or different nations due to varying levels of the factors presented in this paper.

On a final note, we hope that this study provides motivations for further empirical and theoretical research regarding the effect of sharing negative information under various different settings, which can lead to a better understanding of markets under information asymmetry. In particular, empirical research measuring the effect of the factors suggested in our model and analyzing how they affect the inter-industry or inter-country differences in terms of the level of fraud can be one of the important directions for future research.

7 Appendix: Proofs

Proof of Proposition 2: Proof by contradiction. Assume that there are two types that separate: $v_1 < v_2$ with messages $m_1 \neq m_2$. Then these types will set prices $p_1 = v_1$ and $p_2 = v_2$ and will serve the whole market. But that cannot happen since type v_1 would deviate and send the message m_2. Thus at most one type separates. Label this type v_0, and let m_0 be its separating message. After m_0 it will charge the price $p_0 = v_0$ and will serves the whole market. If v_0 is the lowest quality type then we are done. If not, consider type $\nu < v_0$ and denote its sales as s_ν. It cannot be the case that $s_\nu < 1$. Assume not,

$$\begin{aligned}(p_\nu - c_\nu)s_\nu &\geq (p_0 - c_\nu)1 \\ (p_\nu - c_0)s_\nu + (c_0 - c_\nu)s_\nu &\geq (p_0 - c_0)\cdot 1 + (c_0 - c_\nu)\cdot 1 \\ (p_\nu - c_0)s_\nu &\geq (p_0 - c_0)\cdot 1 + (c_0 - c_\nu)(1 - s_\nu) > (p_0 - c_0)\cdot 1,\end{aligned}$$

where the last inequality is strict because $s_\nu < 1$. Then type v_0 would strictly profit from deviating, sending the same message as type ν and charging the same price, p_ν . Furthermore, as p_ν is not necessarily the optimal price for type v_0 his profit from deviation will be even higher.

Thus if v_0 separates $s_\nu = 1$ for any $\nu < v_0$. Consider first the case of loss-averse buyers. Let $m(\nu)$ be the message sent by type ν and $\mu(\nu)$ be corresponding beliefs about the quality. Buyers' utility when purchasing after message ν at price p is $E_{\mu(\nu)}v - p - bELoss_{\mu(\nu)}$. Since all sellers with lower quality, including seller ν, serve the whole market they will charge price $p_\nu = E_{\mu(\nu)}v - B\cdot ELoss_{\mu(\nu)}$, which is the highest price that allows a seller to serve the whole market. Given that neither type v_0 nor type ν want to deviate and mimic each other it must be the case that $p_\nu = p_0$, for any $\nu < v_0$. Thus $E_{\mu(\nu)}v - B\cdot ELoss_{\mu(\nu)} = v_0$ and, therefore, $E_{\mu(\nu)}v \geq v_0$. We can rich the same conclusion for the case of risk-averse buyers. Any seller with $\nu < v_0$ serves the whole market. It will then choose the highest price that allows them to do so: $Eu_{\Gamma,\mu(\nu)}(w + v - p_\nu) = u(w)$, where Γ indicates this is expected utility of the most risk-averse buyer. Since type ν does not mimic type v_0, and vice versa, it has to be the case that $p_\nu = v_0$. But then by Jennsen's inequality $Ev_{\mu(\nu)} > v_0$. The rest of the proof goes regardless of whether buyers are risk- or loss-averse.

So far we established that any $\nu < v_0$ will send an on-equilibrium message that will result in buyers having beliefs $Ev_{\mu(\nu)} > v_0$. We will use it to show that $v_0 \leq Ev$. Let $\mathcal{M}$ be the set of messages sent by sellers with $\nu < v_0$. Let $\mathcal{V}$ be the set of all types that send messages in $\mathcal{M}$. As we established, $E_{\mu(m)}v > v_0$ for any $m \in \mathcal{M}$. Let $\mathcal{V}^c = [v_L, v_H]/(\mathcal{M}\cup\{v_0\})$ be the set of all types that are not in $\mathcal{V}$ or v_0, and $\mathcal{M}^c$ be the set of messages they send. Since all types in $\mathcal{V}^c$ have quality higher than v_0 we have that $E_{\mu(m)}v > v_0$ for any $m \in \mathcal{M}^c$. Thus any equilibrium message results in beliefs about expected quality that are either equal to v_0, which is v_0's separation message, or strictly greater, for all other messages. Given that on-equilibrium beliefs are correct it then has to be the case that $v_0 < Ev$. ■

Proof of Proposition 3: Let μ be beliefs after message m. The buyer indifferent between buying the product and not has the degree of loss-aversion given by

$$E_\mu v - p - b\cdot ELoss_\mu = 0.$$

Then the demand function given beliefs μ can be written as

$$s(\mu,p) = \begin{cases} 0 & \text{if } \dfrac{1}{B}\dfrac{E_\mu v - p}{ELoss_\mu} \leq 0 \\ \dfrac{1}{B}\dfrac{E_\mu v - p}{ELoss_\mu} & \text{if } 0 < \dfrac{1}{B}\dfrac{E_\mu v - p}{ELoss_\mu} < 1 \\ 1 & \text{otherwise} \end{cases}$$

and the corresponding profit is

$$\pi(m,c) = \begin{cases} 0 & \text{if } E_\mu v - c \le 0 \\ \frac{1}{B}\frac{1}{4}\frac{(E_\mu v - c)^2}{ELoss_\mu} & \text{if } 0 < \frac{1}{2B}\frac{E_\mu v - c}{ELoss_\mu} < 1 \\ E_\mu v - B \cdot ELoss_\mu - c & \text{otherwise} \end{cases} \tag{14}$$

The profit function is a differentiable function of cost that consists of three parts. For very large costs it is 0. For intermediate costs it is a quadratic function with derivative wrt cost being between -1 and 0. For small costs it is a straight line with slope of minus 1. Notice that in the small costs' range the straight line given by the third equation in (14) lies below the quadratic expression given by the second line in (14).

First, we show that if v_1 does not serve the whole market then the same holds for any $v > v_1$. That will imply that if there exists $\bar{v}$ that serves the whole market then any $v < \bar{v}$ will also serve the whole market.

Consider type v_1 that sends an on-equilibrium message m and does not serve the whole market given m and his optimal price. Consider type $v > v_1$. If message m is optimal for v then it is trivial that v will not serve the whole market. Both v and v_1 face the same demand function, v_1 chose not to serve the whole market and $c_v > c_{v_1}$. Assume now that for type v message $\hat{m} \neq m$ is optimal, and let $\hat{\mu}$ be beliefs given message $\hat{m}$. It cannot be the case that type v serves the whole market given $\hat{m}$. We show why it is the case in four steps. First, type v_1 weakly prefers m over $\hat{m}$ so that $\pi(m, c_1) \ge \pi(\hat{m}, c_1)$. Second, were v_1 to send $\hat{m}$ then it would also serve the whole market since $c_1 < c_v$. Also, as we already established were v to send m then it would not serve the whole market. Third, then $\pi'_c(\hat{m}, c) = -1$ and $\pi'_c(m, c) > -1$ for any c between c_1 and c_v. Finally, $\pi(m, c_1) \ge \pi(\hat{m}, c_1)$ implies that $\pi(m, c_v) > \pi(\hat{m}, c_v)$ since derivative of the former is higher on interval $[c_1, c_v]$. This is a contradiction to the fact that v prefers message $\hat{m}$.

Second, we show that if types v_1 and v_2 such that $\bar{v} \le v_1 < v_2$ weakly prefer the same message m to any other message. Then any $v \in (v_1, v_2)$ will strictly prefer m over any other message. Assume not. Assume there is type v that weakly prefers message $\hat{m}$. Since $v > \bar{v}$, it does not serve the whole market and his profit from sending message $\hat{m}$ is

$$\pi_{\hat{\mu}}(c) = \frac{(E_{\hat{\mu}} v - c_v)^2}{4B \cdot ELoss_{\hat{\mu}}}.$$

For type v the profit from $\hat{m}$ is weakly higher than the profit from m but for types v_1 and v_2 the profit from m is weakly higher from m than from $\hat{m}$. That would mean that profit functions $\pi(m, \cdot)$ and $\pi(\hat{m}, \cdot)$ intersect at least twice on interval $[c_1, c_2]$ or are tangent at point c_v. We will show that neither is impossible.

Consider the case when $E_\mu v > E_{\hat{\mu}} v$. Let $\kappa(m, c)$ denote the quadratic function that defines profit for intermediate cost range. In general two quadratic functions can intersect at most twice. Parabolas $\kappa(m, c)$ and $\kappa(\hat{m}, c)$ intersect once on interval $[E_{\hat{\mu}} v, E_\mu v]$. This is because $\kappa(m, E_{\hat{\mu}} v) > 0 = \kappa(\hat{m}, E_{\hat{\mu}} v)$ and $\kappa(m, E_\mu v) = 0 < \kappa(\hat{m}, E_\mu v)$. Notice that while $\kappa(m, \cdot)$ and $\kappa(\hat{m}, \cdot)$ intersect on interval $[E_{\hat{\mu}} v, E_\mu v]$ the corresponding profits do not since $\pi(\hat{m}, c) = 0$ and

$\pi(m, c) > 0$ on $[E_{\hat{\mu}}v, E_\mu v)$. Thus we established that $\kappa(m, \cdot)$ and $\kappa(\hat{m}, \cdot)$ intersect at most once outside of $[E_{\hat{\mu}}v, E_\mu v]$. This immediately implies that the quadratic version of profit functions cannot be tangent.[15] Since v_1 and v_2 do not serve the whole market given m and v does not serve the whole market given $\hat{m}$ the profit functions for both coincide with κ functions on interval $[c_1, c_2]$. There is at most one intersection of profit functions on $[c_1, c_2]$, and it is intersection and not tangency. Then it impossible to have that types v_1 and v_2 prefer m, i.e. $\pi(m, c_1) \geq \pi(\hat{m}, c_1)$ and $\pi(m, c_2) \geq \pi(\hat{m}, c_2)$, while for the intermediate type v the deviation profit is weakly higher, $\pi(\hat{m}, c_v) \geq \pi(m, c_v)$. Thus it must be the case that $\pi(\hat{m}, c_v) < \pi(m, c_v)$ The case $E_\mu v < E_{\hat{\mu}}v$ is similar. The case $E_\mu v = E_{\hat{\mu}}v$ is trivial, since by assumption expected losses differ and all types would strictly prefer the message with lower expected loss.

Finally, consider two types $v_1 < v_2 \leq \bar{v}$ that send two different messages m_1 and m_2. We established earlier that both types serve the whole market given their optimal messages. Given beliefs μ, if the sellers chooses the whole market the optimal price to do that is $p_w(\mu) = E_\mu v - B \cdot ELoss_\mu v$. When $p = p_w(\mu)$ the buyer with loss-aversion equal to B is indifferent between purchasing and not. It is the highest price that allows the seller to serve the whole market and any lower price would be suboptimal. In equilibrium it must be the case that $p_w(m_1) = E_{\mu_1}v - B \cdot ELoss_{\mu_1} = E_{\mu_2}v - B \cdot ELoss_{\mu_2} = p_w(m_2)$. If, for example, $p_w(m_1) > p_w(m_2)$ then type v_2 would be better off by sending message m_1 and serving the whole market but with a higher price $p_w(m_1)$. But then it must be the case that sellers v_1 and v_2 are indifferent between m_1 and m_2. ■

Proof of Proposition 4: Consider a Bertrand setting where risk-neutral buyers believe that seller i has expected quality v_i and seller j has expected quality v_j and where sellers' costs are c_i and c_j. Without loss of generality assume that $v_i - c_i \geq v_j - c_j$. Then in the equilibrium seller j charges price $p_j = c_j$ and seller i charges price $p_i = c_j + (v_i - v_j)$ so that $\pi_j = 0$ and $\pi_i = (v_i - c_i) - (v_j - c_j) \geq 0$.

We will apply this observation to our setting. We will show that with two on-equilibrium messages all sellers will strictly prefer the one with a higher expected quality. The proof will be the case when quality distribution is continuous, but it can be easily extended to the discrete distribution case. Assume there is an equilibrium where at least one seller, seller i, sends at least two on-equilibria messages m_i and $\hat{m}_i$ such that $E_{\mu(m_i)}v \neq E_{\mu(\hat{m}_i)}v$. Since $E_{\mu(m_i)}v \neq E_{\mu(\hat{m}_i)}v$ there must be an equilibrium message that results in beliefs above Ev, so wlog we assume that m_i is such a message. Let $m_j(v_j)$ be the messaging strategy of player j.

If type v_i sends a message that results in expected quality ν then its expected profit is

$$\int_{\{v_j : E_{\mu(m_j(v_j))}v - c_j < \nu - c_i\}} ((\nu - c_i) - (E_{\mu(m_j(v_j))}v - c_j))dv_j.$$

For any given $m_j(v_j)$ higher ν generates higher profit for any type v_i. First, the expression inside the integral becomes strictly larger. Second, the integration area weakly expands. Finally, as long as $\nu > Ev$ the integration area has positive measure. The latter follows from the fact that for any

[15] If two different parabolas $ax^2 + bx + c$ and $dx^2 + ex + f$ are tangent at x^0 then $2ax^0 + b = 2dx^0 + e$ for some x^0. But then for any $x > x^0$ the derivative of one parabola is always greater than the derivative of another parabola and so intersection is impossible at $x > x^0$. Same logic to show that no intersection is possible at $x < x^0$.

$m_j(v_j)$ there exists an on-equilibrium message m_j^0 such that $E_{\mu_j(m_j^0)}v \leq Ev$ and, therefore, such that $E_{\mu(m_j^0)}v < E_{\mu(m_i)}v$. Thus for any given $m_j(v_j)$, all types of seller i will strictly prefer message m_i over message $\hat{m}_i$. ■

Proof of Proposition 5: The proof consists of two parts. In the first part we solve a pricing subgame for given messages (m_i, m_j), where message m_i is a separating message sent by type v_i. In the second part we prove the Proposition's statement.

We begin with the first part in which we consider several cases. First case is when seller j also sends a separating message. Second case is when he does not. In the latter case three sub-cases are possible: either both sellers split the market or one of the two sellers serve the whole market. Below we consider each possibility.

Case I: If seller j sends separating message as well then the equilibrium is straightforward. If $v_i - c_i \geq v_j - c_j$ then seller i sets price $p_i = c_j + (v_i - v_j)$ and earns profit $\pi_i = (v_i - c_i) - (v_j - c_j)$, seller j sets $p_j = c_j$ and earns zero profit. If $v_i - c_i < v_j - c_j$ then the outcome is reversed.

Case II: Seller j sends message m_j such that buyers' beliefs about expected quality and expected loss are Ev_j and $ELoss_j > 0$. Three options are possible. Seller j serves the whole market, seller i serves the whole market, or sellers i and j split the market. If sellers split the market, then the preferences of indifferent consumer are

$$v_i - p_i = Ev_j - p_j - b^0 ELoss_j \quad \Rightarrow \quad b^0 = \frac{(Ev_j - v_i) - (p_j - p_i)}{ELoss_j}.$$

Buyers who are more (less) loss-averse than b^0 will purchase product from seller i (j). Demand for seller j is equal to

$$s_j(p_i, p_j) = \begin{cases} \Phi\left(\dfrac{(Ev_j - v_i) - (p_j - p_i)}{ELoss_j}\right) & \text{if } 0 < \dfrac{(Ev_j - v) - (p_j - p_i)}{ELoss_j} < B \\ 1 & \text{if } \dfrac{(Ev_j - v) - (p_j - p_i)}{ELoss_j} \geq B \\ 0 & \text{otherwise} \end{cases}$$

and $s_i(p_i, p_j) = 1 - s_j(p_i, p_j)$.

Case II.a) First we derive conditions for equilibria when sellers do not split the market. An equilibrium where seller j serves the whole market exists if j's best response to $p_i = c_i$ is to set price such that $s_j = 1$.

Let p_j^c be the price such that when seller i charges $p_i = c_i$ and seller j charges p_j^c the value of b^0 is B. From the indifference condition, $p_j^c = (Ev_j - v_i) + c_i - B \cdot ELoss_j$. Price p_j^c is the best response to $p_i = c_i$ when the derivative of j's profit at p_j^c is non-positive. Profit of seller j is $(p_j - c_j)s_j(p_i, p_j)$, and its derivative at (c_i, p_j^c) is

$$-((Ev_j - c_j) - (v_i - c_i) - B \cdot ELoss_j)\phi(B)\frac{1}{ELoss_j} + 1 \leq 0,$$

where 1 in the expression above is $\Phi(B)$. This is equivalent to

$$1 \leq \phi(B)\frac{1}{ELoss_j}((Ev_j - c_j) - (v_i - c_i) - B \cdot ELoss_j). \tag{15}$$

Case II.b) Similarly, let p_i^c be the value of p_i such that if seller j charges price c_j the value of b^0 is 0. From the indifference condition, $p_i^c = c_j - (Ev_j - v_i)$. Equilibrium when seller i serves the whole market exists if i's best response to $p_j = c_j$ is p_i^c, or the derivative of i's profit at p_i^c is non-positive. Seller i's profit is $(p_i - c_i)(1 - s_j(p_i, p_j))$ and its derivative with respect to p_i is equal to

$$-(p_i - c_i)\phi(s_j(p_i, p_j))\frac{1}{ELoss_j} + (1 - \Phi(s_j(p_i, p_j)).$$

Plugging in $p_j = c_j$ and $p_i = p_i^c$ we get

$$((Ev_j - c_j) - (v_i - c_i))\phi(0)\frac{1}{ELoss_j} + 1 \leq 0.$$

For uniform distribution, condition for equilibrium when seller j serves everyone is equivalent to the condition $\dfrac{(Ev_j - c_j) - (v_i - c_i)}{B \cdot ELoss_j} \geq 2$. Condition for equilibrium when seller i serves all the market is equivalent to condition $\dfrac{(Ev_j - c_j) - (v_i - c_i)}{B \cdot ELoss_j} \leq -1$.

Case II.c) Finally, consider the case of interior equilibrium. Under an interior equilibrium p_i and p_j should satisfy the following conditions

$$\begin{cases} b^0 = \dfrac{(Ev_j - v_i) - (p_j - p_i)}{ELoss_j} \\ -\phi(b^0)\dfrac{1}{ELoss_j}(p_i - c_i) + (1 - \Phi(b^0)) = 0 \\ -\phi(b^0)\dfrac{1}{ELoss_j}(p_j - c_j) + \Phi(b^0) = 0 \end{cases}$$

The first equation is the indifference condition, the second equation and third equation are the first-order conditions for sellers i and j respectively. Subtracting the third equation from the second equation we get

$$\frac{\Phi(b^0)}{\phi(b^0)} - \frac{1 - \Phi(b^0)}{\phi(b^0)} = ((p_j - c_j) - (p_i - c_i))\frac{1}{ELoss_j}.$$

It will convenient to denote the LHS of the equation above as $A(b)$. Taking into account the definition of b^0 we can re-write it as

$$b^0 + A(b^0) = ((Ev_j - c_j) - (v_i - c_i))\frac{1}{ELoss_j}. \tag{16}$$

When $\phi(b)$ is a log-concave function then $A(b)$ is an increasing function of b. Indeed $A(b) = \Phi(b)/\phi(b) - (1 - \Phi(b))/\phi(b)$. The first term is an increasing function of b by Theorem 1 in Bergstrom and Bagnoli (2006) and term $(1 - \Phi(b))/\phi(b)$ is a decreasing function of b by Corollary 2 in Bergstrom and Bagnoli (2006). Thus, if an interior equilibrium exists (16) uniquely determines b^0, which then can be used to determine equilibrium prices.[16]. This completes the first part of the proof.

We will not prove the Proposition's statement using our derivations from the first part of the proof. Assume there is an equilibrium where two different types of seller i separate. Denote them

[16]That it is density and not the cdf that is log-concave is required for Corollary 2 but not for Theorem 1

as v_i and v_i' and assume that $v_i > v_i'$. We denote the separating messages of the two types as m_i and m_i'. We will show that for a given m_j of seller j, type v_i' will be strictly better off if it were to send message m_i instead of m_i'. For a given m_j we will denote buyers' beliefs about expected quality of seller j as Ev_j, and buyers' beliefs about expected loss as $ELoss_j$.

Case I: Pricing equilibria given both (m_i, m_j) and (m_i', m_j) are interior. Then b^0 in the former equilibrium satisfies

$$b^0 + A\left(b^0\right) = ((Ev_j - c_j) - (v_i - c_i))\frac{1}{ELoss_j},$$

and b'^0 in the latter equilibrium satisfies

$$b'^0 + A\left(b'^0\right) = ((Ev_j - c_j) - (v_i' - c_i'))\frac{1}{ELoss_j}.$$

Case I.a) If at the messaging stage type v_i' were to deviate and send m_i instead of m_i' then in equilibrium of the post-deviation subgame, if it is interior, b^0_{dev} satisfies

$$b'^0_{dev} + A\left(b'^0_{dev}\right) = ((Ev_j - c_j) - (v_i - c_i'))\frac{1}{ELoss_j}. \tag{17}$$

Here we use the fact that when v_i' sends m_i instead then buyers will beliefs that his quality is v_i. Seller j, knows the true quality and cost of seller i as well as how buyers interpret m_i.

Since $c_i' < c_i$ and since $b + A(b)$ is a strictly increasing function we have that in the new equilibrium $b'^0_{dev} < b^0$. Since $v_i - c_i > v_i' - c_i'$ we have that $b^0 < b'^0$ and, thus, combining the two we get that $b'^0_{dev} < b'^0$. That means that if seller v_i' deviates and mimics type v_i then this seller will increase its sales compared to the original equilibrium. Furthermore, the deviation price will go up as well. Indeed, comparing the equilibrium and the deviation scenarios we have

$$\frac{p_i' - c_i'}{ELoss_j} = \frac{1 - \Phi(b'^0)}{\phi(b'^0)} < \frac{1 - \Phi(b^0_{dev})}{\phi(b^0_{dev})} = \frac{p'_{dev} - c_i'}{ELoss_j}.$$

As mentioned earlier, term $(1 - \Phi(b))/\phi(b)$ is a decreasing function of b and $b'^0_{dev} < b'_0$. Thus $p_i' < p'_{dev}$. By mimicking type v_i, type v_i' will get a higher market share and a higher price. Thus, in this subgame mimicking v_i' will be a profitable deviation.

Case I.b) Consider the case when type v_i' mimics type v_i but, differently from Case Ia), the deviating equilibrium is not an interior. Then it means that in the deviating subgame seller i serves the entire market. The deviation equilibrium price is $p'_{dev} = p_i^c$, where p_i^c was defined in Case IIb) of the first part of the proof, and the the derivative of seller v_i''s profit is non-positive at p_i^c. Thus

$$\frac{p'_{dev} - c_i'}{ELoss_j} \geq \frac{1 - \Phi(0)}{\phi(0)} > \frac{1 - \Phi(b'^0)}{\phi(b'^0)} = \frac{p_i' - c_i'}{ELoss_j}.$$

After deviation type v_i' increases its market share to one and charges a higher price. Thus its deviation profit is strictly higher.

Case II: Consider the case when one of the pricing subgames has a corner equilibrium, and seller i seller i serves in that equilibrium serves the whole market. Since $v_i - c_i > v_i' - c_i'$ it cannot be the case that in subgame (m_i', m_j) seller i serves the whole market but in subgame (m_i, m_j) seller i does not. That follows from the condition on the corner equilibrium derived in the first part.

Consider now the (m_i, m_j)-equilibrium type v_i serves the whole market. If in (m_i', m_j) equilibrium type v_i' does not serve the whole market, it will obviously find m_i to be strictly more profitable. Message m_i increases its market share to 1 and it will be able to charge high price:

$$\frac{p'_{dev} - c'}{ELoss_j} \geq \frac{1 - \Phi(0)}{\phi(0)} > \frac{1 - \Phi(b'^0)}{\phi(b'^0)} = \frac{p_i' - c_i'}{ELoss_j}.$$

If in both (m_i, m_j) and (m_i', m_j) equilibria seller i serves the whole market, type v_i' will have incentives to deviate because the post-deviation market share will stay equal to 1, but the post-deviation price will increase: $p'^c_{i,dev} = c_j - (Ev_j - v_i) > c_j - (Ev_j - v_i') = p'^c_i$.

<u>Case III:</u> Consider the case when in one of the pricing equilibria seller j serves the whole market. It can only happen at the (m_i', m_j) subgame. Then if in (m_i, m_j) seller i makes positive sales, type v_i' will strictly benefit from deviating, since it will earn positive, and not zero profit. The only instance when type v_i' does not strictly gain from mimicking type v_i is if in both (m_i, m_j) and (m_i', m_j) seller j serves the whole market and in the post-deviation equilibrium seller j still serves the whole market as well. In this instance both m_i and m_i' bring zero profit against seller j that sent message m_j.

Case III is the only case which allows for a possibility of type v_i' to be indifferent between m_i and m_i'. This possibility can only realize if v_i and v_i' earn zero profit against a given seller j. In all other cases seller's v_i' preferences for m_i is strict. Thus to complete the proof it is enough to show that in any symmetric equilibrium the expected profit of v_i' is positive, which would imply that mimicking v_i is strictly profitable. When the quality distribution is discreet this point is trivial. With a positive probability quality of seller j is equal to v_i and, since the equilibrium is symmetric, seller j sends message m_i. Type v_i earns zero profit in this case as the Bertrand equilibrium ensues. Type v_i', however, if it deviates and sends message m_i will earn positive profit since buyers' beliefs are identical but type v_i' has a lower cost.

Now consider the case when the quality distribution is continuous and both v_i and v_i' earn zero expected profit in a symmetric equilibrium. It will be convenient to generalize (15) to the case of any buyers' beliefs. If (m_i, m_j) results in beliefs $(Ev_i, ELoss_i)$ and $(Ev_j, ELoss_j)$ then there is a pricing equilibrium where seller j serves the whole market if

$$1 \leq \phi(B)\frac{1}{ELoss_j}((Ev_j - B \cdot ELoss_j - c_j) - (Ev_i - B \cdot ELoss_i - c_i)). \tag{18}$$

Among messages sent by types $v > v_i'$ pick the message m with the highest $Ev_m - B \cdot ELoss_m$. Since $v_i' < v_i$ there exist types with $v > v_i'$. If type v_i' sends message m and quality of seller j is above v_i' then inequality (18) cannot be satisfied. It means that by sending message m type v_i' will earn positive profit in any pricing game against every seller j with quality above v_i'. Since there is a positive measure of sellers with quality above v_i' it means that message m is a profitable deviation as it results in positive expected profit for type v_i'. Thus there is no equilibrium where v_i' earns zero expected profit. ■

Proof of Proposition 6: In what follows we will proceed with the case when the quality distribution is continuous as the case of discreet quality distribution is similar. Furthermore, we only look at the case when pricing equilibria are interior, as the corner equilibria can be handled in a similar way to the loss-averse buyers case.

If seller v_i separates and beliefs about seller j are given by density function f_j then the indifference condition for γ^0 is

$$\int e^{-\gamma^0(v_j-v_i)} f_j(v_j)dv_j = e^{-\gamma^0(p_j-p_i)}. \tag{19}$$

Taking logarithm of both sides and dividing by minus γ^0 we get

$$-v_i - \frac{1}{\gamma^0}\ln\left(\int e^{-\gamma^0 v_j} f_j(v_j)dv_j\right) = p_j - p_i, \tag{20}$$

and notice that γ^0 is fully determined by price difference $p_i - p_j$.

Using (20), we can re-write the maximization problem of firm i, $\max_{p_i}(p_i-c)(1-\Phi(\gamma^0(p_i-p_j)))$, so that i's choice variable is its market share and not the price:

$$\max_{\gamma}\left(v_i - c_i + p_j + \frac{1}{\gamma}\ln\left(\int e^{-\gamma v_j} f_j(v_j)dv_j\right)\right)(1-\Phi(\gamma)). \tag{21}$$

It is equivalent to the original maximization problem in that for a given p_j the optimal p_i will result in the same γ as the solution to (21) given p_j. From (21) follows that if (p_i^*, p_j^*) is an equilibrium of the subgame where seller i has quality v_i and cost c_i then $(p_i^* + \kappa, p_j^*)$ is an equilibrium of the subgame where seller i has quality $v_i + \kappa$ and cost $c_i + \kappa$ since seller i's best response does not change. Furthermore, the profits of both sellers is the same in both equilibria.

Using this observation we can now prove the proposition. The proof is by contradiction. Assume that there are two types $v_i > v_i'$ that separate with messages m_i and m_i'. We will show that for any pricing subgame type v_i' will strictly prefer sending message m_i, which means that m_i' was not optimal.

Lemma 1 *Let m_j be the message sent by seller j. Assume that $c_i > c_i'$. If pricing equilibria when seller i's quality/cost is (v_i, c_i) and when it is (v_i', c_i') are interior than the former is less profitable for seller i.*

Proof. Let p_j^* and $p_j'^*$ denote j's equilibrium prices given c_i and c_i' respectively. We will show that $c_i - c_i' > p_j^* - p_j'^*$. Assume not. First, consider the case when $c_i - c_i' = p_j^* - p_j'^*$. If that were the case then it follows from (21) that seller i would find it optimal to choose the same γ^0 in both cases. That would mean that $p_i^* - p_i'^* = p_j^* - p_j'^*$. From the fact that p_j^* satisfies the FOC given p_i^* and that fact that $p_j^* > p_j'^*$ follows that seller j would be better of charging a higher price than $p_j'^*$ when seller i charges $p_i'^*$:

$$0 = -\phi(\gamma^0)\frac{\partial\gamma^0(p_i^* - p_j^*)}{\partial p_i}(p_j^* - c_j) + \Phi(\gamma^0) < -\phi(\gamma^0)\frac{\partial\gamma^0(p_i'^* - p_j'^*)}{\partial p_i}(p_j'^* - c_j) + \Phi(\gamma^0).$$

The first equality is that the derivative of π_j when $p_i = p_i^*$ and $p_j = p_j^*$ is equal to zero. It holds because (p_i^*, p_j^*) is an equilibrium. The third term is the derivative of π_j when $p_i = p_i'^*$ and $p_j = p_j'^*$.

It is less than the second terms because $p_j'^* - c_j$ is less than $p_j^* - c_j$ and because γ is an increasing function of p_i so its derivative is positive. Thus $p_j'^*$ is not best response to $p_i'^*$, and $(p_i'^*, p_j'^*)$ cannot be an equilibrium of the pricing stage.

Case when $c_i - c_i' < p_j^* - p_j'^*$ is similar. When $c_i - c_i' < p_j^* - p_j'^*$ it follow from (21) that it is optimal for seller i to set price so that $\gamma'^0 > \gamma^0$. Denote this price as $p_i'^*$. Consider price p_j'' such that $\gamma^0 = \gamma(p_i'^* - p_j'') = \gamma(p_i^* - p_j^*)$ and note that $p_j'' > p_j'^*$ because $\gamma^0 < \gamma'^0$ (recall that γ is an increasing function of the price difference, $p_i - p_j$). By the logic similar to the $c_i - c_i' = p_j^* - p_j'^*$ case, the derivative of π_j at $(p_i'^*, p_j'')$ is positive, which means that the price that sets it to zero must be greater than p_j'' and, therefore, greater than $p_j'^*$.[17] Which means that $(p_i'^*, p_j'^*)$ is not an equilibrium.

Thus we established that $c_i - c_i' > p_j^* - p_j'^*$, which implies that $p_j^*(c_i) - c_i$ is a decreasing function of c_i. Then applying envelope theorem to (21) we immediately get that the equilibrium profit of firm i is a decreasing function of c_i. That completes the proof of the lemma. ■

By the observation above if we compare a pricing equilibrium where seller i has quality/cost (v_i', c_i') with a pricing equilibrium where seller i has quality/cost $(v_i, c_i' + (v_i - v_i'))$ then both equilibria result in the same profits as well as p_j^*. As for p_i^* it will increase by exactly the same amount as the cost increase, which is $(v_i - v_i')$.

Now we can complete the proof. By Lemma 1 a pricing equilibrium where seller i's quality/cost is (v_i, c_i') is more profitable than when it is (v_i, c_i), which in turn is more profitable than the pricing equilibrium where seller i's quality/cost is $(v_i, c_i' + (v_i - v_i'))$ (by Lemma 1 and because $v_i - c_i > v_i' - c_i'$). Finally, pricing equilibrium $(v_i, c_i' + (v_i - v_i'))$ is as profitable as (v_i', c_i'), which follows from (21). Combining all the steps we get that the pricing equilibrium (v_i, c_i') is more profitable than (v_i', c_i') and therefore type v_i' will be better off by deviating and mimicking type v_i. As mentioned earlier, the case when of the equilibria is not interior is similar to the loss-averse case. This completes the proof of the theorem.■

Proof of Proposition 7: Notice that the indifference condition (7) is always satisfied when $\gamma = 0$. This is because $u(x) \equiv 1$ when $\gamma = 0$ and the indifference condition becomes trivial.

Let y denote $e^{-\gamma^0}$ so that (7) is

$$y^{p_H - p_L} - q_{LH} - (1 - q_{LH}) y^{v_H - v_L} = 0. \tag{22}$$

As γ varies between 0 and $+\infty$, variable y varies between 1 and 0. There is one solution $y = 1$, which corresponds to $\gamma = 0$. In order to prove the Proposition, we need to show that on interval $0 \le y < 1$ there is at most one solution. Given the root $y = 1$, it is equivalent to showing that there are at most two solutions of (22) on $0 \le y \le 1$.

Assume not. If function (22) has three or more roots on $[0, 1]$ then its derivative should have two or more roots on $[0, 1]$. Taking derivative of the LHS of (22) with respect to y and setting it

[17] We use here that the profit function is concave function of p_j, so that its derivative decreases. A sufficient condition for that it that $\Phi(\gamma)$ is sufficiently concave.

equal to zero we get:

$$(p_H - p_L)y^{p_H - p_L - 1} - (1 - q_{LH})(v_H - v_L)y^{v_H - v_L - 1} = 0,$$

so that

$$1 - (1 - q_{LH})\frac{v_H - v_L}{p_H - p_L}y^{(v_H - v_L)-(p_H - p_L)} = 0.$$

Clearly, the equation above has at most one solution on interval $[0, 1]$. Therefore, equation (22) has at most two solutions on $[0, 1]$. As one solution is $y = 1$ it implies that there is at most one solution when $0 < y < 1$.

To prove the second part of the proposition, we observe that buyers prefer the L-product if

$$1 - e^{-\gamma(v_L - p_L)} \geq q_{LH}(1 - e^{-\gamma(v_L - p_H)}) + (1 - q_{LH})(1 - e^{-\gamma(v_H - p_H)}),$$

which is equivalent to

$$e^{-\gamma(p_H - p_L)} \leq q_{LH} + (1 - q_{LH})e^{-\gamma(v_H - v_L)}. \tag{23}$$

When $\gamma = +\infty$ then (23) is satisfied as its LHS is zero, and the RHS is positive. In other words, extremely risk-averse buyers will always purchase the L-product. Thus if $\gamma^0 > 0$ is the solution to (7) then all types with $\gamma < \gamma^0$ purchase the H-product and all types with $\gamma > \gamma^0$ purchase the L-product. ■

Proof of Proposition 8: The proof of the Proposition is fairly long and it consists of two parts. In the first part we reduce the equilibrium system (11) to one equation with one unknown, γ^0. In the second part, we analyze that equation and develop sufficient conditions on $\Phi(\gamma)$ stated in Proposition 8. We begin the first part of the proof proving Lemma 2 which says for the purpose of proving the existence one can ignore the last equation in (11) and an unknown variable λ.

Lemma 2 *If for a given $q_{LH} \in [0, 1]$ the solution $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ to the reduced system exists, then there exists $q \in [0, 1]$ and $\lambda \in [0, 1]$ such that $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H, \lambda)$ is a solution to the equilibrium system (11).*

Proof. Plug values $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ into the the last equation of (11) to recover q_L. Notice that for any $(\bar{\gamma}^0, \bar{p}_L, \bar{p}_H)$ one can find q_L such that it is between 0 and 1 and the last equation is satisfied. From q_L and q_{LH} one can then uniquely recover q and λ: $q = (1 - q_{LH})(1 - q_L)$ and $\lambda = q_L/(1 - q)$. The only thing one has to check is that $\lambda, q \in [0, 1]$ and that $q_{LH} \leq 1 - q$. The last inequality states that share of low-quality sellers who announce high quality, q_{LH}, cannot be higher than the total share of low quality sellers, $1 - q$. This is straightforward. That $q_{LH} \leq 1 - q$ is trivial:

$$q_{LH} \leq 1 - q = 1 - (1 - q_{LH})(1 - q_L) = q_{LH} + q_L - q_{LH} \cdot q_L.$$

Given that $q = (1 - q_{LH})(1 - q_L)$ is it between 0 and 1. Finally, $\lambda < 1$ is equivalent to $q_L < 1 - q$, which is also true.

$$q_L \leq 1 - q = 1 - (1 - q_{LH})(1 - q_L) = q_{LH} + q_L - q_{LH} \cdot q_L.$$

This completes the proof of the Lemma. ■

In what follows, we will call (11) without the last equation as the reduced system. The reduced system has three equations, three unknowns (γ^0, p_L, p_H), and it treats q_{LH} as a given parameter. The three equations of the reduced system are the indifference condition

$$e^{-\gamma^0(p_H-p_L)} = q_{LH} + (1-q_{LH})e^{-\gamma^0(v_H-v_L)}, \tag{24}$$

and two FOCs that determine prices:

$$\begin{cases} -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_L + (1-\Phi(\gamma^0)) = 0 \\ -\phi(\gamma^0)\dfrac{\partial\gamma^0(p_L,p_H)}{\partial p_L}p_H + \Phi(\gamma^0) = 0 \end{cases} \tag{25}$$

Take the second equation of (25) and subtract from it the first equation of (25) we get

$$\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \frac{1-\Phi(\gamma^0)}{\phi(\gamma^0)} = \frac{\partial\gamma^0(p_L,p_H)}{\partial p_L}(p_H-p_L).$$

Just like in the proof of Proposition 5 it will be convenient to denote $\dfrac{\Phi(\gamma^0)}{\phi(\gamma^0)} - \dfrac{1-\Phi(\gamma^0)}{\phi(\gamma^0)}$ as $A(\gamma^0)$. From (24) we get that

$$\begin{aligned} \frac{\partial\gamma^0}{\partial p_L} = -\frac{\partial\gamma^0}{\partial p_H} &= -\frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{-(p_H-p_L)e^{-\gamma^0(p_H-p_L)} + (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}} = \\ &= \frac{\gamma^0 e^{-\gamma^0(p_H-p_L)}}{(p_H-p_L)e^{-\gamma^0(p_H-p_L)} - (1-q_{LH})(v_H-v_L)e^{-\gamma^0(v_H-v_L)}}. \end{aligned}$$

Let $\Delta p = p_H - p_L$ and $\Delta v = v_H - v_L$. Thus the reduced system becomes a system of two equations and two unknowns:

$$\begin{cases} e^{-\gamma^0\Delta p} &= q_{LH} + (1-q_{LH})e^{-\gamma^0\Delta v} \\ A(\gamma^0) &= \dfrac{\gamma^0 e^{-\gamma^0\Delta p}}{\Delta p e^{-\gamma^0\Delta p} - (1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}\Delta p \end{cases} \tag{26}$$

Next we solve for Δp from the second equation of (26):

$$A(\gamma^0) = \frac{\gamma^0 e^{-\gamma^0\Delta p}}{\Delta p e^{-\gamma^0\Delta p} - (1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}\Delta p.$$

We can re-write it as

$$\begin{aligned} A(\gamma^0)\Delta p e^{-\gamma^0\Delta p} - A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v} &= \gamma^0\Delta p e^{-\gamma^0\Delta p} \\ A(\gamma^0)\Delta p - A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v}e^{\gamma^0\Delta p} &= \gamma^0\Delta p \\ A(\gamma^0)(1-q)\Delta v e^{-\gamma^0\Delta v}\frac{e^{\gamma^0\Delta p}}{\Delta p} &= A(\gamma^0) - \gamma^0 \\ \frac{\gamma^0 A(\gamma^0)(1-q_{LH})\Delta v e^{-\gamma^0\Delta v}}{A(\gamma^0)-\gamma^0} &= \gamma^0\Delta p e^{-\gamma^0\Delta p}. \end{aligned} \tag{27}$$

It is straightforward to show that in equilibrium $\Delta p > 0$. Indeed, consider the indifference condition

$$e^{-\gamma^0(p_H - p_L)} = q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)},$$

and take natural logarithm of both sides:

$$-\gamma^0(p_H - p_L) = \ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)}).$$

The expression inside the logarithm is less than one. Then $\ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0(v_H - v_L)}) < 0$, which implies that $\Delta p > 0$. From second and third equations of (11) then it follows that $\frac{\Phi(\gamma^0)}{\phi(\gamma^0)} > \frac{1 - \Phi(\gamma^0)}{\phi(\gamma^0)}$ and, therefore, in equilibrium $A(\gamma^0) > 0$. Furthermore, since $\Delta p > 0$ the LHS of (27) must be positive. Since, in equilibrium $A(\gamma^0) > 0$ it then implies that $A(\gamma^0) > \gamma^0$. If such γ^0 does not exist then (27) cannot be satisfied and no equilibrium with the disclosure of negative information can exist.

We can now eliminate Δp from (27). From the indifference condition we get that

$$\gamma^0 \Delta p = -\ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0 \Delta v}),$$

and then

$$\gamma^0 \Delta p e^{-\gamma^0 \Delta p} = -\ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0 \Delta v})(q_{LH} + (1 - q_{LH})e^{-\gamma^0 \Delta v}).$$

Plugging it into (27) we get that the equilibrium value of γ^0 is determined by

$$-\frac{\gamma^0 A(\gamma^0)(1 - q_{LH})\Delta v e^{-\gamma^0 \Delta v}}{A(\gamma^0) - \gamma^0} = (q_{LH} + (1 - q_{LH})e^{-\gamma^0 \Delta v}) \ln(q_{LH} + (1 - q_{LH})e^{-\gamma^0 \Delta v}). \tag{28}$$

This completes the first part of the proof. In the second part of the proof, we will analyze equation (28) and derive condition that determine whether the solution exists or not. In what follows we will refer to the RHS and LHS of (28) as simple RHS and LHS without referring to the equation's number.

Proof of i and ii:) As we have established earlier if $A(\gamma) < \gamma$ for every γ then the solution to (27) and, therefore, to (28) does not exist. We will show that this is the case for uniform and convex distributions. Let the support of $\Phi(\gamma)$ be $[0, \Gamma]$. For uniform and convex distributions it is finite. Inequality $A(\gamma) < \gamma$ is equivalent to $2\Phi(\gamma) - 1 < \gamma\phi(\gamma)$. We will write it as $\Phi(\gamma) - 1 < \gamma\phi(\gamma) - \Phi(\gamma)$. Function $\Phi(\gamma)$ is weakly convex function such that $\Phi(0) = 0$. By a standard property of convex functions $\gamma\phi(\gamma) \geq \Phi(\gamma) - \Phi(0)$ and since $\Phi(0) = 0$ we have $\gamma\phi(\gamma) \geq \Phi(\gamma)$. Thus in the inequality $\Phi(\gamma) - 1 < \gamma\phi(\gamma) - \Phi(\gamma)$ the left-hand side is negative and the right-hand side is non-negative, which means that it is satisfied for any $\gamma \in [0, \Gamma)$. When $\gamma = \Gamma$ and $\Phi(\gamma)$ is linear then $A(\Gamma) = \Gamma$, in all other cases $A(\Gamma) < \Gamma$. Thus, for the case of convex and uniform distribution functions there is no equilibrium where both firms split the market.

Proof of iii:) The RHS is a continuous function of γ. It is negative for any $\gamma > 0$. When $\gamma = 0$ it is equal to zero. When $\gamma \to \infty$, its limit is equal to $q_{LH} \cdot \ln(q_{LH}) < 0$.

The LHS is discontinuous when $A(\gamma)=\gamma$. Let $\hat{\gamma}$ denote the largest root such that $A(\gamma)=\gamma$. We can show that it exists. First, $A(0)=-1/\phi(0)<0$. Second, $\lim_{\gamma\to\infty}\gamma\phi(\gamma)=0$. If the limit is positive, say $z>0$, than it means that for all sufficiently large γ^0, say for all $\gamma>\Gamma^0$, it has to be the case that $\phi(\gamma)>\frac{1}{2}\frac{z}{\gamma}$. But then

$$\int_{\Gamma^0}^{\infty}\phi(s)ds>\frac{1}{2}\int_{\Gamma^0}^{\infty}\frac{z}{\gamma}d\gamma=\infty,$$

which is a contradiction since it has to be less or equal than 1. Third,

$$\lim_{\gamma\to\infty}(A(\gamma)-\gamma)=\lim_{\gamma\to\infty}\frac{2\Phi(\gamma)-1-\gamma\phi(\gamma)}{\phi(\gamma)}=\frac{1}{0}=\infty.$$

Given that $A(\gamma)-\gamma$ is continuous we can conclude now that it has roots and that the there is the largest root. In other words, there exists $\hat{\gamma}$ such that $A(\hat{\gamma})=\hat{\gamma}$ and $A(\gamma)>\gamma$ for every $\gamma>\hat{\gamma}$. Therefore, the LHS is a continuous function for any $\gamma>\hat{\gamma}$.

We can now prove the equilibrium existence. Since $\hat{\gamma}$ is the largest root it means that for any $\gamma>\hat{\gamma}$ it must be the case that $A(\gamma)>\gamma$, and in a sufficiently small right neighborhood of $\hat{\gamma}$ fraction $A(\gamma)/(A(\gamma)-\gamma)$ is close to plus infinity. Then the LHS is close to $-\infty$ and, therefore, is less than the RHS. When γ is close to infinity, the LHS gets arbitrarily close to zero. This is because all terms of the LHS, including $A(\gamma)/(A(\gamma)-\gamma)$, are bounded and the term $e^{-\gamma\Delta v}$ converges to zero. That $A(\gamma)/(A(\gamma)-\gamma)$ is bounded follows from

$$\lim_{\gamma\to\infty}\frac{A(\gamma)}{A(\gamma)-\gamma}=\frac{2\Phi(\gamma)-1}{2\Phi(\gamma)-1-\gamma\phi(\gamma)}=1.$$

Therefore, for sufficiently large γ the LHS of (28) is less than the RHS. By continuity the solution to (28) exists.

Proof of iv:) Let support of $\Phi(\gamma)$ be $[0,\Gamma]$ where $\Gamma<\infty$. Then $A(\Gamma)>\Gamma$. Indeed, $A(\Gamma)>\Gamma$ is equivalent to $1>\Gamma\phi(\Gamma)$. Assume that it is not satisfied so that $\phi(\Gamma)>1/\Gamma$. Then, since $\phi(\gamma)$ is strictly decreasing we have

$$1=\int_0^{\Gamma}\phi(s)ds>\int_0^{\Gamma}\frac{1}{\Gamma}ds=1,$$

which is a contradiction. Also, one can show that $A(\gamma)<\gamma$ when γ is sufficiently close to zero, or more precisely for any γ such that $\Phi(\gamma)<1/2$. Thus there exists γ such that $A(\gamma)=\gamma$ and let $\hat{\gamma}$ be the largest such γ. Then the LHS of (28) is continuous when $\gamma\in(\hat{\gamma},\Gamma]$. As in case iii), one could try to use continuity to establish that the solution to (28) exists. However, it might not work with the original distribution because unless Γ is sufficiently large, the LHS will not be close enough to zero to guarantee that the solution exist.

Consider now a cdf function Φ_α defined as $\Phi(\alpha\gamma)$. It is a concave function with support $[0,\Gamma/\alpha]$. Now the largest value of γ is Γ/α. By taking α sufficiently small we can make the support $[0,\Gamma/\alpha]$ large enough so that $\gamma e^{-\gamma\Delta v}$ can be made sufficiently close to zero within the support.

Term $A(\gamma)/(A(\gamma)-\gamma)$ on the other hand will not change. Let $A_\alpha(\gamma)$ be defined similarly to $A(\gamma)$ but with a cdf Φ_α. Then for any $\gamma \in [0,\Gamma]$,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha)-\gamma/\alpha} = \frac{A(\gamma)}{A(\gamma)-\gamma}.$$

Indeed,

$$\frac{A_\alpha(\gamma/\alpha)}{A_\alpha(\gamma/\alpha)-\gamma/\alpha} = \frac{2\Phi_\alpha(\gamma/\alpha)-1}{2\Phi_\alpha(\gamma/\alpha)-1-(\gamma/\alpha)\phi_\alpha(\gamma/\alpha)} = \frac{2\Phi(\gamma)-1}{2\Phi(\gamma)-1-(\gamma/\alpha)\alpha\phi(\gamma)} = \frac{A(\gamma)}{A(\gamma)-\gamma}.$$

Thus, when α is sufficiently small we can apply the reasoning of case iii) to function $\Phi_\alpha(\gamma)$ to show that the solution exists. ■

Proof of Proposition 9: The proof is based on the Lemma below.

Lemma 3 *Assume that price p results in beliefs $\mu(p)$ and sales s, while price p' results in beliefs μ' and sales s'. Assume type v weakly prefers price p to price p'. Then*

i) if $s < s'$ then all higher-cost types ($\nu < v$) will strictly prefer p to p';

ii) if $s > s'$ then all lower-cost types ($\nu < v$) will strictly prefer p to p'.

Proof. Prove part i) first. Let $\nu > v$. Since type v weakly prefers p to p' it follows that

$$\begin{aligned}(p-c_v)s &\geq (p'-c_v)s' \\ (p-c_\nu)s+(c_\nu-c_v)s &\geq (p'-c_\nu)s'+(c_\nu-c_v)s' \\ (p-c_\nu)s &\geq (p'-c_\nu)s'+(c_\nu-c_v)(s'-s) > (p'-c_\nu)s',\end{aligned}$$

where the last inequality follows from the fact that $c_\nu > c_v$ and $s' > s$.

Part ii) is similar. Let $\nu < v$ and then

$$\begin{aligned}(p-c_v)s &\geq (p'-c_v)s' \\ (p-c_\nu)s+(c_\nu-c_v)s &\geq (p'-c_\nu)s'+(c_\nu-c_v)s' \\ (p-c_\nu)q(p) &> (p-c_\nu)q(p)+(c_v-c_\nu)(s'-q(p)) \geq (p'-c_\nu)s'.\end{aligned}$$

It completes the proof. ■

First, we show that if there are two types $v_1 < v_2$ that on-equilibrium choose the same price p with a positive probability then all types in (v_1, v_2) will strictly prefer p over any other price and, therefore, will choose it with probability 1. Assume not. Assume type $v' \in (v_1, v_2)$ prefers price $p' \neq p$. If price p' results in higher sales then by the first part of Lemma 3 type v_2 would also strictly prefer p' to p and, therefore, would not choose p with positive probability. Similarly, if price p' results in lower sales then by the second part of Lemma 3 type v_1 would strictly prefer p' to p and, therefore would not choose p with positive probability. Finally, consider the case when price p' results in the same sales as p. Then if $p' > p$ both v_1 and v_2 would strictly prefer p' over p and would not choose p with positive probability. Similarly, if $p' < p$ then no type would choose price p', including type v'. This proves the interval structure of the equilibrium as defined in i). To show indifference in the case of continuous quality distribution, consider type v_i where $1 < i < N+1$. If type v_i strictly prefers message p_i to p_{i-1} then all types in a sufficiently small neighborhood of v_i also strictly prefer p_i. Similarly if type v_i strictly prefers p_{i-1} to p_i then all all types in a sufficiently small neighborhood of v_i will strictly prefer m_{i-1}. Contradiction.

Next we show the monotonicity of prices and sales. Consider type v_i that is indifferent between (p_{i-1}, s_{i-1}) and (p_i, s_i). If the quality distribution is discreet than v_i does not necessarily belong to its support, however, it exists which will suffice for our purpose. It can't be the case that $s_i = s_{i-1}$ because $p_i \neq p_{i-1}$. It cannot be that $s_i < s_{i-1}$ either. Were it the case then by Lemma 3 all types with $v < v_i$ would strictly prefer p_i to p_{i-1}, which contradicts the definition of v_i. Thus, $s_i > s_{i-1}$. But then $p_i < p_{i-1}$ because were it not the case all the types in $[v_{i-1}, v_i]$ would strictly prefer price p_i and enjoy both higher price and higher sales. ■

Proof of Proposition 10: Proof by contradiction. Assume there are at least three different on-equilibrium prices: $p_1 > p_2 > p_3 > \ldots$. Let p_1 be the price set by seller v_L. Then $p_1 \leq E_{p_1} v$ as otherwise seller v_L would not make any sales. By Proposition 9 then $p_i < E_{p_i} v$ for any i, which means that the corresponding sales must be equal to 1 for every $i \neq 1$. When buyers are risk-neutral, it is strictly optimal for each buyer to purchase a product when a price is strictly below expected quality. But, as we showed in Proposition 9, one cannot have an equilibrium where two different prices result in the same sales. Thus, there are most two different equilibrium prices. One is the price chosen by low types, including v_L, and that results in $s_1 < 1$, and the second price that is chosen by by all other types and that results in $s_2 = 1$.

It is straightforward to construct an equilibrium with two on-equilibrium prices. Assume there are two types: $\{L, H\}$ with $v_L = 1, c_L = 1$ and $v_H = 2, c_H = 1/2$. The probability of type H is $2/3$. Two equilibrium prices are $p_1 = 3/2$ and $p_2 = 1$. Type L chooses p_1 with probability 1 and type H randomizes between p_1 and p_2 with equal probabilities. Buyers beliefs about expected quality are $\mu(p_1) = 3/2$ and $\mu(p_2) = 2$. The corresponding sales are $q_1 = 1/2$ and $q_2 = 1$. One can see that H sellers in indifferent between p_1 and p_2 and seller L strictly prefers p_1. Off-equilibrium beliefs to support this equilibrium are: $\mu(p) = 1$ if $p \neq 1$ and $p \neq 3/2$.■

Proof of Proposition 11: Fix N. Assume that quality is distributed uniformly on $[1, N+1]$. We will specify sellers' costs as well as the distribution of buyers' preferences so that there exists an equilibrium where for any integer i types in $[i, i+1]$ pool. When types $[i, i+1]$ pool and set price p_i the expected loss is equal to

$$\int_i^{i+1} (v - Ev) dv = \int_i^{i+1} \left(v - \frac{i+i+1}{2} \right) dv = \frac{1}{8},$$

and the corresponding demand is given by the location of the indifferent buyer, $Ev - p - b\frac{1}{8}$, so that

$$q(p) = \frac{1}{B} 8 \left(\frac{i+i+1}{2} - p \right).$$

Take p_1 as any number between 1 and 1.5 and $p_N = 1$. Here 1.5 is the expected quality of sellers who pool using price p_1, and 1 is the lowest possible quality. Let $\epsilon = (p_1 - p_N)/(N-1)$

and $p_i = p_1 - (i-1)\epsilon$. Type $i+1$ should be indifferent between p_i and p_{i+1}. We will use this indifference to determines c_{i+1}:

$$8((2i+3)/2 - p_{i+1})(p_{i+1} - c_{i+1}) = 8((2i+1)/2 - p_i)(p_i - c_{i+1}).$$

Taking into account that $p_i - p_{i+1} = \epsilon$ we get

$$c_{i+1} = \frac{p_{i+1} - \frac{2i+1}{2}\epsilon + \epsilon(p_{i+1} + p_i)}{1+\epsilon}.$$

Note that since prices decrease with i we get that cost decreases as well so that $c_i > c_{i+1}$. The expression above only determines costs for types with integer quality. For types in between integer points one can use any strictly decreasing function $c(v)$ as long as $c(i) = c_i$ and $c(i+1) = c_{i+1}$. Finally, the corresponding sales are

$$s_{i+1} = \frac{1}{B}\left(\frac{2i+3}{2} - (p_1 - (i-1)\epsilon)\right),$$

and they are increasing function of i. Set B so that $s_N = 1$.

Finally, off-equilibrium beliefs are such that if $p \neq p_i$ for some i then buyers believe that the seller has the lowest quality of 1.

We can now verify that it's an equilibrium. Any deviation to an off-equilibrium price $p > 1$ will result in zero sales and zero profit. Any deviation to $p < 1$ will result in sales of 1. However, price $p_N = 1$ also result in sales of 1 but at a higher price and, therefore, is more profitable. Deviations to on-equilibrium prices are not profitable by Proposition 9. Consider, for example, sellers with $v \in (i, i+1)$. By construction, type $i+1$ is indifferent between p_{i+1} and p_i. From Proposition 9 follows that $s_{i+1} > s_i$ and that all types with $v > i+1$ strictly prefer to p_{i+1} to p_i while all types with $v < i+1$ strictly prefer to p_i. By applying same logic for all types with integer j one can establish that that all types with $v \in (i, i+1)$ strictly prefer price p_i over any other equilibrium price.■

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