Optimal Stress Tests and Diversification*

Keshav Dogra†
Federal Reserve Bank of New York

Keeyoung Rhee‡
Korea Development Institute

January 31 2017

Abstract

We present a model to study whether a regulator should reveal imperfect information about banks' financial health or not. Stress tests can restore market confidence in banks. However, the regulator's information is inevitably imperfect, thereby misclassifying some financially sound banks as risky. To avoid being misclassified, banks will choose portfolios that the regulator deems safe, making the financial system less diversified than without the stress tests. As a consequence, while it is always ex post optimal to reveal stress test results, this policy is not ex ante optimal. We show that the ex-ante optimal policy is non-monotonic in the underlying economic state: information should be revealed either in very good times or in very bad times.

JEL classification: D82, G11, G18

Keywords: Stress tests, optimal disclosure, portfolio diversification

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of New York, the Federal Reserve System, or Korea Development Institute. Heehyun Lim provided able research assistance.

†Keshav.Dogra@ny.frb.org

‡ky.rhee829@gmail.com

1 Introduction

Since the 2008 financial crisis, stress tests have become a key tool of supervisory policy. The 2009 Supervisory Capital Assessment Program (SCAP) tested whether the largest U.S. bank holding companies would have adequate capital in the event of a severe recession. The bank-specific results of the test, including projected losses by asset type and capital shortfalls, were disclosed to the public. One goal of stress tests is to reduce investors' uncertainty about banks' future losses and capital requirements, making them more willing to invest; and indeed, several authors credit the SCAP with restoring confidence in the financial system (Bernanke (2013); Morgan, Peristiani and Savino (2014)). Following the success of the SCAP, the Dodd-Frank Act requires the Federal Reserve to conduct annual stress tests of large financial institutions.

But stress tests are inevitably imperfect, because regulators have limited information about both the risks facing the economy, and the exposure of particular asset classes to these risks.[1] Frame, Gerardi and Willen (2015) present a cautionary example; the Office of Federal Housing Enterprise Oversight (OFHEO) conducted a risk-based capital stress test for government-sponsored enterprises (GSE) such as Fannie Mae and Freddie Mac, but it failed to detect the GSEs' risk and insolvency. Frame, Gerardi and Willen (2015) show that the OFHEO was too optimistic, both about aggregate risk (their adverse house price scenario was less severe than the actual housing bust) and about Fannie and Freddie's exposure to this risk (their model substantially underpredicted defaults).

Because stress test results are imperfect, releasing them may induce banks to make inefficient investment decisions. Since banks may be incorrectly classified as 'risky' by the regulator, they may choose their portfolios to avoid this penalty. As a result, banks may not make investment decisions based on their own information. Rather, they may withdraw their (possibly superior) model of measuring financial risk and adapt to the regulator's model. Such a "model monoculture" may not only lead to inefficient investments, but also induce banks to herd in financial markets, increasing systemic risks.[2]

We present a model to ask whether stress tests can have these unintended consequences;

[1] In his speech at the "Maintaining Financial Stability: Holding a Tiger by the Tail" financial markets conference, Ben Bernanke pointed out another limit of stress tests, namely that they struggle to measure bank-specific risk:

> Another challenge is that our stress scenarios cannot encompass all of the risks that banks might face. For example, although some operational risk losses, such as expenses for mortgage put-backs, are incorporated in our stress test estimates, banks may face operational, legal, and other risks that are specific to their company or are otherwise difficult to estimate.

[2] Gillian Tett in the Financial Times claimed that routinization of stress tests may induce banks to share a similar view on how to measure and manage their financial risks, but regulators may not take this possibility into account:

> The point about a model monoculture is that it makes risk models pretty useless. As Donald MacKenzie of Edinburgh University notes, what models cannot measure is the chance that banks all act as a herd – creating financial panics.

and, if so, how the regulator should optimally disclose stress test results. In the model, outside investors are fully rational, risk averse, and their incentives are aligned with the banks. Banks choose ex ante whether to invest in a 'good' project, or a 'diversifying' project, which pays off in a different state of the world. There are also a random measure of 'bad' banks, who act mechanically, mimicking good and diversifying banks in order to borrow from outside investors and invest in unproductive projects. To initiate projects, banks need to borrow from outside investors in a capital market, but they face an adverse selection problem. Specifically, the outside investors cannot observe which type each bank is – whether it is good, diversifying, or bad – but only know the fraction of each type present in the market.

A key assumption is that a regulator can provide additional – but imperfect – information to the investors. Specifically, we assume that the regulator can correctly identify good banks, but cannot distinguish between diversifying and bad banks. The regulator can disclose its superior information when the banks go to the market to fund their projects, but such disclosure has different effects on adverse selection for 'good' and 'diversifying' banks. On the one hand, the regulator's information fully separates the good banks from the others – in particular, from bad banks – and removes the adverse selection problem faced by good banks. Thus good banks can successfully fund their projects in the capital market. On the other hand, the diversifying banks are lumped together with only the bad banks. Consequently, if the population of the bad banks is large, the diversifying banks may not be able to borrow in the capital market due to the severe adverse selection problem.

We analyze what the regulator's optimal disclosure policy will be from both an ex ante and an ex post perspective. We first show that if the regulator ex post decides whether to release its superior information, it is always (weakly) socially desirable to publicize the information. If there are few bad banks in the market, adverse selection problems will be mild, whether the information is released or not, so both 'good' banks and 'diversifying' banks can borrow in capital markets. By contrast, suppose there are so many bad banks that good and diversifying banks cannot finance their projects in the market. In this case, the regulator can improve social welfare by releasing its superior information, because this removes the lemons problem faced by the good banks, restoring their market access.

However, if the regulator can commit, before the banks choose projects, whether to share its information in particular states of the world, it is not always optimal to share information. Releasing the information only alleviates the adverse selection problem faced by 'good' banks, and therefore encourages most banks to invest in the 'good' project ex ante. This under-diversified portfolio exposes the economy to a higher risk – in the event that the 'good' project fails – than would obtain without the regulator's information, thereby reducing ex-ante social welfare.

Our main finding is that the ex-ante optimal disclosure policy is non-monotonic with respect to economic conditions, captured by the population of bad banks in the market. Specifically, we show that the regulator should commit to release its superior information if and only if the adverse selection problem at the capital market is either relatively mild (the fraction of the 'bad' banks is below a threshold) or relatively severe (the fraction of the bad banks is above another threshold). The regulator commits to release information in states of the world where adverse selection problems are severe, in order to partially activate the capital market for the 'good' banks. However, such a disclosure policy benefits only the 'good' banks, which inefficiently increases banks' choice of the 'good' projects. To mitigate this problem, the regulator refrains from releasing its information in states with moderate adverse selection problems.

Interestingly, it is also optimal for the regulator to disclose its information in states of the world where there are very few bad banks in the market. In these states, telling investors that a bank is not 'good' does not necessarily imply that the bank is 'bad'; instead, it is very likely to be 'diversifying'. After receiving the regulator's information, the investors prefer to fund these 'not good' banks because 'diversifying' banks are relatively scarce (relative to an equilibrium without any information revelation). Knowing this, the regulator commits to release its information in the states with mild adverse selection problems, as a way to increase diversification.

1.1 Related literature

A growing theoretical literature discusses the benefits and costs of disclosing stress test results. Goldstein and Sapra (2013) and Leitner (2014) survey four arguments against disclosure of stress test results: full disclosure of stress test results may reduce risk-sharing among financial firms a la Hirshleifer (1971) (Goldstein and Leitner, 2015); forcing firms to disclose their financial status too often may encourage short-termism (Gigler et al., 2014); public information provided by stress tests may crowd out private information held by individual creditors (Morris and Shin, 2002); and a regulator's disclosure of stress tests may adversely affect its ability to learn from the market because the public information reduces incentives for investors in the market to acquire private information, making market prices less informative (Bond and Goldstein, 2015). We complement this literature by focusing on a different potential cost of stress tests, namely that by misclassifying some banks, they may undermine diversification.

Our finding that disclosure is optimal in a crisis, but not in normal times, is shared with a number of recent papers which arrive at this result for different reasons. (Goldstein and Leitner, 2015) also find that no disclosure is optimal in normal times, because of the Hirshleifer effect: while the financial system has enough capital to share risk, revealing banks' liquidity shocks would preclude insurance against such risks. Partial disclosure is optimal in a crisis, because banks do not have enough capital to share risk. Bouvard, Chaigneau and Motta (2015) study how disclosure affects financial stability when banks face rollover risk. Disclosure is desirable in a crisis, because it can prevent bank runs, but undesirable in normal times, because it can cause bank runs. These papers argue that disclosure is not always optimal ex post, taking the distribution of banks' 'type' as given. Our contribution is to ask how the anticipation of stress tests might change banks' investment in projects ex ante. To this end, we present a model in which disclosure is always optimal ex post, and argue that it may not be optimal ex ante, since it reduces diversification.

While our main result - that releasing public information reduces diversification - may seem reminiscent of Morris and Shin (2002), the mechanism in our paper is different. In Morris and Shin (2002), public information reduces diversification because agents' actions are strategic complements, and they coordinate on the public signal. In our paper, banks have no intrinsic coordination motive. If anything, they have an intrinsic motive for diversification, because risk-averse households reward banks for investing in assets that pay off when consumption is low. However, disclosure of public information causes outside investors to rationally punish banks for investing differently from the herd, because those banks may be risky.

Our results also relate to the recent empirical literature on stress tests. Morgan, Peristiani and Savino (2014) and Flannery, Hirtle and Kovner (2015) present evidence that the Supervisory Capital Assessment Program (SCAP) stress tests in 2009 were informative: banks with larger capital gaps experienced more negative unexpected returns. This empirical finding is consistent with one of the ex-post optimality of full revelation of the regulator's information in our model. On the contrary, Glasserman and Tangirala (2015) document that routinized stress tests – such as the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act Stress Testing (DFAST) program – become less informative over time. Their empirical evidence suggests that unconditional releasing of the regulator's information may not be effective as intended, consistent with our main finding that the ex-ante optimal disclosure is non-monotonic to the severity of the adverse selection problem in the market.

Some researchers analyze influences of other policy instruments on the effectiveness of bank stress tests. Spargoli (2013) argues that disclosure of negative stress test results either reduces lending or requires costly bailouts. Faria-e Castro, Martinez and Philippon (2015) make a similar point, arguing that disclosure can create inefficient bank runs, unless there is a fiscal backstop.

2 Model

2.1 Setup

Consider a three-period economy ($t = 0, 1, 2$) in which *(i)* a continuum of banks can undertake long-term financial projects which create stochastic returns at $t = 2$; *(ii)* a continuum of investors fund those projects by purchasing bonds issued by the banks; *(iii)* a regulator, who seeks to maximize social welfare, decides whether or not to reveal information about banks at date 1. Nature randomly draws an underlying state of the economy $\omega \in \{G, D\}$ where $\omega = G$ with probability $p \in (0, 1)$. No one in this economy knows what the true state is until date 2.

At $t = 0$, a continuum of banks with measure one individually choose which project to undertake. Each individual bank can choose one of two projects $\theta \in \{\gamma, \delta\}$: project γ returns one unit of cash flow at $t = 2$ if $\omega = G$ while it returns nothing if $\omega = D$; project δ returns one unit of cash flow at $t = 2$ only in the state $\omega = D$. After the bank makes its selection, a continuum of 'bad' banks enter into the economy. Their projects – henceforth called type β – return nothing in any state to the economy. However, bad banks earn private benefits by undertaking their projects which do not contribute to social welfare. The fraction of these bad banks $b > 0$ is a random variable with distribution $F(\cdot)$ over support $[0, \infty)$. The true value of b is drawn after the banks other than the bad ones choose their projects. For convenience of exposition, each bank's financial project is hereafter referred as its type.

At $t = 1$, the banks – including bad ones – must raise $x > 0$ dollars in order to continue their projects. They raise these funds by issuing bonds to a continuum of outside investors with measure one at the bond market. The outside investors compete for funding the banks a lá Bertrand by offering repayment terms R at $t = 2$ per unit of capital.

There are several assumptions on the agents' preferences and the information structure. First, every investor exhibits the same expected utility function $U(\cdot)$ as follows:

$$U(c_1, c_2) = c_1 + \mathbb{E}[u(c_2)] = c_1 + pu(c_G) + (1 - p)u(c_D),$$

where c_1 is consumption at $t = 1$, c_G is consumption in the state $\omega = G$ at $t = 2$, and c_D is consumption in the state $\omega = D$ at $t = 2$. Throughout the paper, we assume $u(x) = \log x$; we show in section 4 that our results are not altered if we assume CRRA utility. Since investors are risk averse, they have an incentive to purchase bonds issued by both types of bank in order to smooth consumption across states G and D. Second, we assume that every investor owns an equal amount of shares issued by all banks. This implies that social welfare is simply the aggregated expected utility of the investors. Third, except the bad banks, there is no moral hazard between the banks and the shareholders in this economy: either a type γ bank or a type δ bank behaves in the best interest of its shareholders. Fourth, we assume that each bank's type is private information, which creates an adverse selection problem at the bond market.

Finally, the key assumption is that there exists a regulator who maximizes social welfare – i.e., the investors' expected utility – by choosing whether to reveal his superior but imperfect information about the banks' private types at date 1, before the bond market opens. More specifically, we assume that the regulator can distinguish type γ banks from the other types, but he cannot distinguish type δ banks from type β banks. The regulator receives this information before the banks enter the bond market. Moreover, he can commit whether or not to disclose its information, contingent on the realization of b, before banks choose their type.

2.2 Interpretation

Type γ banks represent financial institutions that are investing in socially desirable, low-risk projects that can be identified as such by the regulator's stress test models. Type δ banks represent institutions that are investing in *different* desirable, low-risk projects. These projects are especially socially desirable - if most banks are type γ - because they diversify the economy's aggregate portfolio. Crucially, we assume that type δ cannot be identified as low-risk by the regulator's stress test models. The justification for this is that regulators' risk models are necessarily imperfect.

Bad banks are introduced in order to create an adverse selection problem, so that there is a potential benefit to releasing stress test results in order to prevent adverse selection and market breakdown. However, releasing stress test results also has an ex ante cost, in that it increases the relative cost to banks of investing in socially valuable projects that are not identified as such by regulators.

One interpretation of these assumptions is that the regulator understands that two kinds of crisis could occur: state G, and state D. She understands relatively well how economic and financial market variables would evolve in state G, and can project how the value of bank assets, loan charge-offs, and so on, would behave in this state. In particular, looking at a particular bank, the regulator can judge whether or not it would have adequate capital in state G. In contrast, while the regulator knows that *other* kinds of crises could occur (state D), she does not know how aggregate variables, and particular banks' portfolios, would behave in these crises.

The regulator can run a stress test based on scenario G. Doing so would distinguish the banks who would perform well in this scenario from the other. However, the banks who 'fail' this stress test do so for different reasons. Some of them ('diversifying' banks) would perform well in the other, less well-understood kinds of crisis. But some of them ('bad' banks) simply have risky portfolios that would perform badly in any adverse scenario.

An alternative interpretation is that the regulator understands how macroeconomic and financial market variables will behave in a crisis, but is uncertain about how particular asset classes would evolve in this crisis. The best the regulator can do is to identify a class of assets which, in her judgment, will perform well with probability p. She expects that with probability $1 - p$, her judgment will turn out to be incorrect, and some other assets will pay off. However, she cannot distinguish the 'diversifying' assets – which would perform well if her model is wrong – from the 'bad' assets, which will perform badly in any crisis.

3 Partial equilibrium

Throughout the paper, we restrict our attention to a symmetric equilibrium in which *(i)* every bank chooses type γ with probability $g \in [0, 1]$; *(ii)* every investor enters into the bond market with probability $\phi \in [0,1]$. In this section, we describe the partial equilibrium at $t = 1$ to analyze how investors and banks strategically interact in the bond market, given the measure of banks with type γ projects g, δ projects $d = 1 - g$, and β projects b.

Bank decisions. Type γ and δ banks enter date 1 with pre-existing projects of total size 1. A type γ bank needs to invest $x < 1$ new funds in order to produce 1 unit of output in state G at date 2, and similarly δ-banks must invest x to produce one unit of output in state D. Thus the return on investment is $\bar{R} := 1/x > 1$.

Investor decisions. Due to asymmetric information, the investors do not know whether a bank is type γ, δ, or β. Once entering the bond market, the investors meet type-γ banks with probabilities $\pi_g = \dfrac{g}{g+d+b}$ and type-δ banks with probability $\pi_d = \dfrac{d}{g+d+b}$, respectively.

At the bond market, the outside investors fund the banks in the following mechanism: *(i)* each investor, indexed by $i \in [0,1]$, quotes an order (q_i, R_i) where the investor purchases q_i units of bonds for repayment terms R_i per unit of capital; *(ii)* the banks sell their bonds to the investors offering the lowest repayment terms; *(iii)* if there are banks who are not funded by the investors quoting the lowest repayment, they go to the investors offering the second lowest repayment, and continue to search for funding to the next lowest repayment until the bond market clears. The bond market clears if all banks sell their bonds (i.e., $\int_0^{1+b} q_i di \geq 1 + b$) or all the quoted buy orders are met (i.e., $\int_0^{1+b} q_i di < 1 + b$). In this mechanism, one can show that no equilibrium other than the symmetric equilibrium (i.e., $(q_i, R_i) = (q, R)$ for all $i \in [0,1]$) exists.[3] Throughout our paper, we restrict our attention to a (partial) symmetric equilibrium in which each investor purchases $q = \phi(1 + b)$ bonds issued by banks with the unit repayment R. Note that the variable ϕ precisely captures a fraction of the banks funded at the market.

Given the total repayment terms Rx, each investor solves the following funding decision problem:

$$\max_{\phi \in [0,1]} -\phi x(1+b) + pu(y_G + (1+b)\pi_g Rx\phi) + (1-p)u(y_D + (1+b)\pi_d Rx\phi),$$

[3] To see this, suppose first there is an equilibrium in which $R_i > R_j$ for some investors $i, j \in [0,1]$ who purchase the bonds as they quote. Given $R = R_i, R_j$, the equilibrium quantities of bond purchase $q_i = q(R_i)$ and $q_j = q^*(R_j)$ are uniquely determined by the following utility maximization problem:

$$q^*(R) = \arg\max_{q \geq 0} \mathbb{E}[u(y_\omega + q_i Rx)] - qx,$$

where y_ω is the total dividends of the banks equally distributed to the investors in each state $\omega \in \{G, D\}$. Since $R_i > R_j$, $\lim_{y\to 0} u'(y) = \infty$, $\lim_{y\to\infty} u'(y) = 0$, and $u'' < 0$, we have

$$\mathbb{E}[u(y_\omega + q_i R_i x)] - q_i x > \mathbb{E}[u(y_\omega + q_j R_j x)] - q_j x.$$

Thus the investor j deviates and mimics the investor i, a contradiction.

Next suppose there are relatively few banks sellable to investor i so that she buys the bonds in a smaller quantity than $q^*(R_j)$ in equilibrium. Then we have either $\mathbb{E}[u(y_\omega + q_i R_i x)] - q_i x \geq \mathbb{E}[u(y_\omega + q_j R_j x)] - q_j x$ or $\mathbb{E}[u(y_\omega + q_i R_i x)] - q_i x \leq \mathbb{E}[u(y_\omega + q_j R_j x)] - q_j x$, in either case one investor deviates and mimics the other investor, a contradiction.

where y_ω is a state-dependent dividend equally distributed to each investor for every state $\omega \in \{G, D\}$. The investor (weakly) prefers to go into the bond market – i.e., $\phi \geq 0$ – if and only if

$$1 \leq [pu'(c_G)\pi_g + (1-p)u'(c_D)\pi_d]R.$$

If the inequality is strict, then $\phi = 1$. If the inequality does not hold, then $\phi = 0$. Dividends distributed to the investors in states G, D are $y_G = [1 - R^*x]\pi_g(1+b) = [1 - R^*x]g$, $y_D = [1 - R^*x]\pi_d(1+b) = [1 - R^*x]d$ respectively. Consumption in the two states must be equal to the total returns of the banks' projects, thus we have $c_G = g$ and $c_D = d$.

3.1 Equilibrium Structure

We consider an equilibrium with the following structure:

Definition 3.1. *An equilibrium is R, ϕ such that given g, d, and b,*

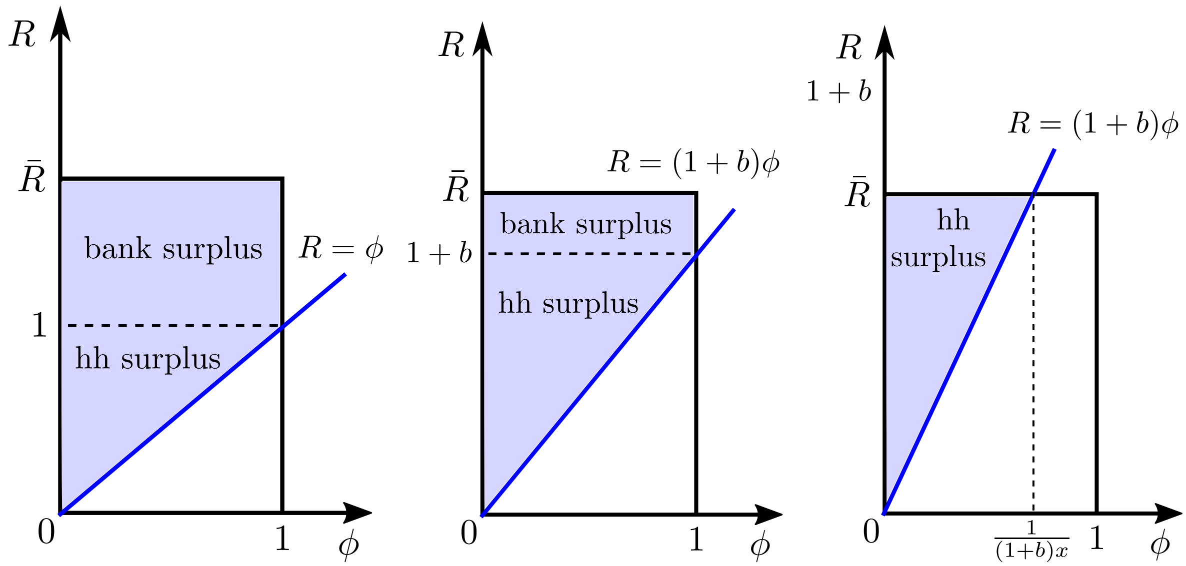
$$\frac{1}{1+b} = [pu'(\phi g)\pi_g + (1-p)\pi_d u'(\phi d)]R,$$
$$\begin{cases} \phi \in [0,1) & \text{if } Rx = 1, \\ \phi = 1 & \text{if } Rx < 1. \end{cases}$$

This equilibrium concept captures credit rationing in the capital market caused by adverse selection. If the fraction of bad banks in the market is sufficiently small, investors will demand only a small spread in order to hold bonds, and the interest rate will be low enough that banks have some equity in their project at date 2, after repaying bondholders: $R \leq \overline{R} \equiv 1/x$. As a result, all investors will enter the capital market and compete to obtain funding of their project. However, if there are too many bad banks, investors will demand such a high spread ($R > \overline{R}$) to compensate them for the risk of funding a bad bank that banks would have no equity left in the market after borrowing at these rates. If investors charged such a high interest rate, good and diversifying banks would drop out of the market, leaving only bad banks. As good and diversifying banks drop out of the market, investors become poorer at date 2, and become willing to lend to banks at a lower interest rate. In equilibrium, $R = \overline{R}$ and lending to banks is rationed, so that each bank can obtain funding with probability $\phi \in (0,1)$.

Lemma 3.2. *If $pu'(g)g + (1-p)gu'(d) \geq x(g+d+b)$, $\phi = 1$ and $R = [pu'(g)\pi_g + (1-p)\pi_d u'(d)]^{-1}$.*
If $pu'(g)g + (1-p)gu'(d) < x(g+d+b)$, ϕ is the solution to

$$pu'(\phi g)g + (1-p)u'(\phi d)d = x(g+d+b),$$

and $R = [pu'(g)\pi_g + (1-p)\pi_d u'(d)]^{-1}$.



(a) $b = 0$ **(b)** Low b **(c)** High b

Figure 1 Equilibrium in the bond market

For the remainder of this section, unless otherwise specified, we specialize to the case of log utility:

Assumption 3.3. *(i)* $u(c) = \ln c$*; (ii)* $p > x$*; and (iii)* $g + d = 1$.

To simplify notation, in what follows we denote $b_x := x(1+b)$.

Lemma 3.4. *Suppose Assumption 3.3 holds. Then*

(i) If $b_x \leq 1$, $\phi = 1$, $R = b_x/x$.

(ii) If $b_x > 1$, $R = 1/x$, $\phi = 1/b_x$.

Ex-ante welfare is

$$U_0(g, b_x) = pu(\phi g) + (1-p)u(\phi d) - \phi x(1+b) = w(g) + \ln \phi - \phi x(1+b) = w(g) - \min\{b_x, 1 + \ln b_x\},$$

where we define $w(g) := p \ln g + (1-p)\ln(1-g)$.

Three subfigures in Figure 1 illustrate the equilibrium in the bond market for different values of b. In all cases, the supply of bonds - measured by ϕ, the share of banks that can be financed - is an increasing function of the interest rate. The demand for credit is a decreasing function of the interest rate. When rates are low, banks each inelastically demand to refinance all their projects. When $R = \bar{R} = 1/x$, demand is perfectly elastic: banks are indifferent between funding and not funding. If $R > \bar{R}$, demand for credit is zero, as the interest rate is higher than the internal rate of return.

In the leftmost figure, there are no bad banks, $b = 0$. Moving to the middle panel, as we increase b, the slope of the bond supply function becomes steeper, as investors demand a higher interest rate to compensate them for the risk of lending to a bad bank. This increases equilibrium interest rates. Eventually, interest rates increase all the way to $\bar{R}$. Above this point, further increases in b are lead to a reduction in credit rather than an increase in interest rates, as shown on the rightmost figure. In this region, a higher b leads to more rationing, and a smaller ϕ. For any finite b, though, banks are able to obtain some financing.

3.2 The Ex-Post Optimal Information Disclosure

We now suppose that the regulator reveals her information to the public. Recall that the regulator can tell apart γ-type banks from others, while she cannot distinguish δ-types from β-types. The following assumption ensures that it would be efficient to finance type γ projects.

Assumption 3.5. *$p\ u'(g) > x$*

Investors can now perfectly identify γ banks, while there are a remaining set of banks that could be either type δ or type β. Thus investors can now hold two distinct securities. They can invest in γ banks, receiving R_g in state G, and nothing in state D. Or they can invest in the other banks, receiving $\pi_d R_d$ in state D (by the law of large numbers argument described above). Now the fraction of δ banks in the second market is $\dfrac{d}{d+g+b}$.

Date 1 equilibrium is defined as follows.

Definition 3.6. *An equilibrium with stress tests is R_g, R_d, ϕ_1 such that, given g, d, b, $\pi_d = \dfrac{d}{d+b}$,*

$$\begin{gathered} 1 = pu'(g)R_g, \\ 1 = (1-p)u'(\phi d)\pi_d R_d, \\ \begin{cases} \phi_1 \in [0,1) & \text{if} \quad R_d x = 1, \\ \phi_1 = 1 & \text{if} \quad R_d x < 1. \end{cases} \end{gathered}$$

The following Lemma states that releasing the signal increases rationing for those banks who are rationed.

Lemma 3.7. *$\phi_1 \le \phi_0$ for every $\beta \ge x$, where inequality strictly holds for $\phi_0 < 1$.*

Proof. In the rationing regime with no signal, ϕ_0 is defined by

$$pu'(\phi_0 g)g + (1-p)u'(\phi_0 d)d = b_x$$

In the rationing regime with a signal, ϕ_1 is defined by

$$(1-p)u'(\phi_1 d)d = b_x - xg$$

Define $\phi(\alpha)$ by

$$(1-\alpha)pu'(\phi(\alpha)g)g + (1-p)u'(\phi(\alpha)d)d = b_x - \alpha xg$$

By definition, $\phi(0) = \phi_0, \phi(1) = \phi_1$. By the Implicit function theorem,

$$\phi'(\alpha) = \frac{[pu'(\phi(\alpha)g) - x]g}{(1-\alpha)pu''(\phi(\alpha)g)g^2 + (1-p)u''(\phi(\alpha)d)d^2} < 0$$

So $\phi_1 < \phi_0$: when there is rationing in both regimes, there is more rationing (lower ϕ) with the signal. □

Lemma 3.8. *Suppose Assumption 3.3 holds. Then*

(i) If $b_x \le \breve{b}_x(g) := 1 - p + xg$, $\phi_1 = 1$.

(ii) If $b_x > \breve{b}_x(g)$, $\phi_1 = \dfrac{1-p}{b_x - xg}$.

The ex-ante welfare is

$$U_1(g, b_x) = w(g) + (1-p)\ln\phi_1 - xg - \phi_1 x(d+b) = w(g) - \min\left\{b_x, 1 - p + xg + (1-p)\ln\left(\frac{b_x - xg}{1-p}\right)\right\}.$$

The following Lemma states formally that it is always ex post optimal to reveal the signal.

Lemma 3.9. *We have the following properties of U:*

(i) U_0, U_1 are strictly decreasing in g when $g > g^*$.

(ii) For the difference $U_1(g, b_x) - U_0(g, b_x)$, we have

$$\text{For all } \; b_x \le \breve{b}_x(g), \; U_1(g, b_x) - U_0(g, b_x) = 0$$
$$\text{For all } \; b_x > \breve{b}_x(g), \; U_1(g, b_x) - U_0(g, b_x) > 0 \;\text{ and }\; \frac{\partial}{\partial b_x}[U_1(g, b_x) - U_0(g, b_x)] > 0.$$

Proof. (i) When $g > g^*$,

$$\frac{\partial U_0}{\partial g} = w'(g) < 0,$$
$$\frac{\partial U_1}{\partial g} = \begin{cases} w'(g) < 0 & \text{if } \; b_x < \breve{b}_x(g), \\ w'(g) - x\left[1 - \dfrac{1-p}{b_x - xg}\right] < 0 & \text{otherwise.} \end{cases}$$

(ii) When $b_x \leq \check{b}_x(g)$, $U_1 - U_0 = 0$. When $b_x \in [\check{b}_x(g), 1]$,

$$U_1(g, b_x) - U_0(g, b_x) = b_x - xg - 1 - p - (1-p)\ln\left(\frac{b_x - xg}{1-p}\right),$$
$$\implies \frac{\partial[U_1(g, b_x) - U_0(g, b_x)]}{\partial b_x} = 1 - \frac{1-p}{b_x - xg} > 0.$$

When $b_x > 1$,

$$U_1(g, b_x) - U_0(g, b_x) = p + \ln b_x - xg - (1-p)\ln\left(\frac{b_x - xg}{1-p}\right),$$
$$\implies \frac{\partial[U_1(g, b_x) - U_0(g, b_x)]}{\partial b_x} = \frac{1}{b_x} - \frac{1-p}{b_x - xg} > 0,$$

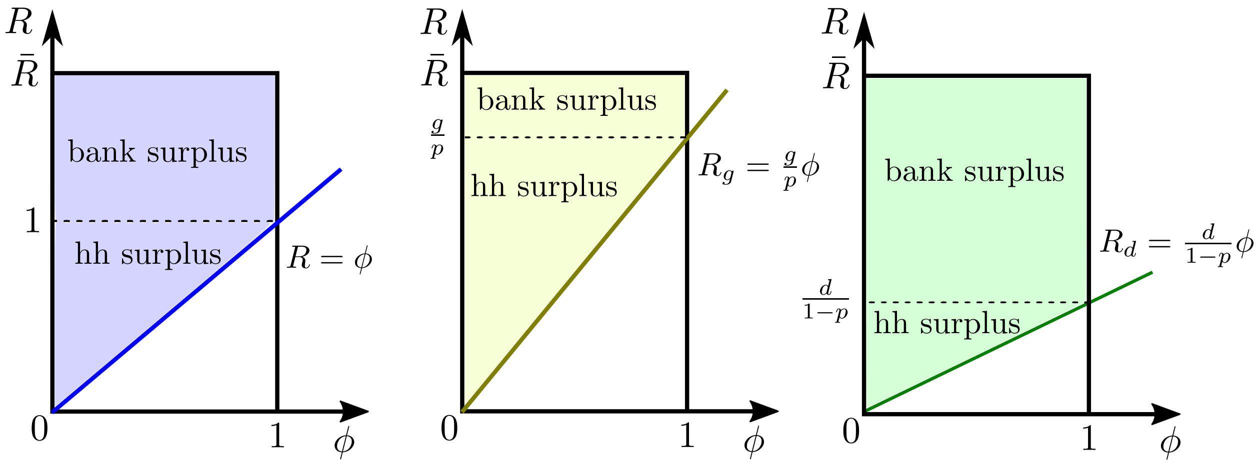
where the last inequality holds because $b_x > 1 > xg/p$. So $U_1(g, b_x) - U_0(g, b_x) > 0$ for $b_x > \check{b}_x(g)$. □

The effect of releasing the signal depends on the number of bad banks in the market, and also on banks' prior investments in g and d projects.

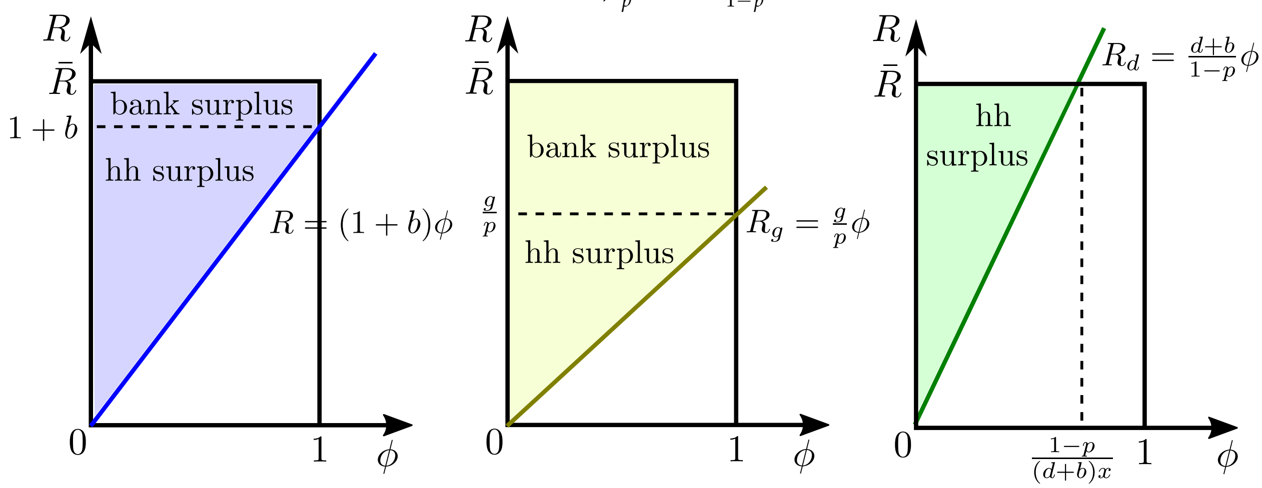
The simplest case to consider is where there are no bad banks ($b = 0$) and investment is efficient ($g = p, d = 1 - p$). In this case, the fair price of γ and δ bank debt is the same, and is equal to 1. Thus releasing the signal does not change market equilibrium in any way. Each bank can still borrow at $R = 1$.

Next, suppose that there are still no bad banks ($b = 0$), but there is inefficiently high investment in g banks: $\frac{g}{p} > 1 > \frac{d}{1-p}$. In this case, δ banks would have a higher fair value than γ banks, and would be able to borrow at a lower interest rate, under full information. This is because investors are poorer in state D, and value assets that pay off in this state more highly. Figure 2a illustrates the effect of releasing the signal in this case. The leftmost panel shows the equilibrium in the integrated market in the absence of information. Once information is released, the market segments. The middle panel shows equilibrium in the market for γ banks. The supply curve shifts up, and equilibrium interest rates are higher for g banks. The rightmost panel shows the equilibrium for δ banks. Their supply curve shifts down, and their equilibrium interest rates are lower. In this environment, then, releasing the signal benefits δ banks at the expense of γ banks. Note, however, that social surplus is unchanged.

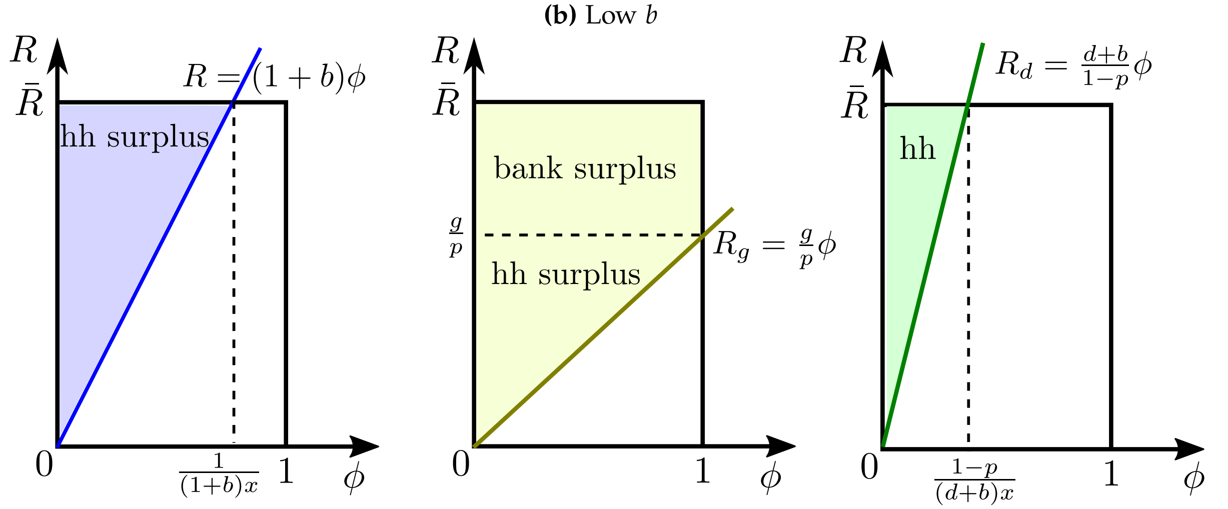
Next, suppose there are some bad banks ($b > 0$), but not so many that there is rationing in the integrated bond market ($(1 + b)x < 1$). Figure 2b illustrates the effect of releasing the signal in this case. Again, the leftmost panel shows the equilibrium without the signal, and the middle and rightmost panels show the equilibrium for G and D banks, respectively, if the signal is released. In this case, the supply curve for G banks shifts down, as investors learn that these banks are definitely not type B. At the same time, the supply curve for banks who receive a negative signal shifts up, as investors believe it is more likely that these banks are type B. In



(a) $b = 0$, $\frac{g}{p} > 1 > \frac{d}{1-p}$



(b) Low b



(c) High b

Figure 2 Equilibrium with and without disclosure the case illustrated, this upward shift is large enough that it leads to rationing in the market for type D banks. Social surplus increases on average, since releasing the signal reduces inefficient investment in bad banks. In addition, the surplus of G banks increases, while the surplus of D banks falls to zero.

Finally, Figure 2c illustrates a case in which there are so many bad banks that there is rationing in the integrated bond market ($(1+b)x > 1$). In the integrated equilibrium, shown in the leftmost panel, all banks are rationed and make zero surplus. After revelation of the signal, the supply curve of G banks shifts down, and these banks make positive surplus; the supply curve of D banks shifts up, and these banks are rationed further, and still face zero surplus. Again, social surplus increases on average. In this case, revealing the signal prevents rationing, at least for some banks. Note that while D banks are made no worse off by revelation, G banks are made strictly better.

Summing up: if the regulator shares her information with the public, this affects both bank profits and social welfare. Releasing the signal has no effect on welfare when b is low, and always increases welfare when b is high, as it prevents investment in bad banks, and increases investment in at least some good banks. When b is low, releasing the signal benefits D banks, if there is inefficiently low investment in these banks (or G banks, if there is inefficiently low investment in these banks). However, when b is high, releasing the signal always harms D banks, relative to G banks. This will affect ex ante investment, to which we now turn.

4 The Full Equilibrium Analysis

4.1 The Ex-Ante Bank Decision

As described above, at date 0, a continuum of banks with measure 1 choose to be either γ or δ. The measure of b_x banks b - or equivalently, $b_x = x(1+b)$ - follows a distribution $F(b_x)$, and is realized at $t = 1$. The policymaker commits at date 0 to an *information policy*, denoted by a set B: the policymaker reveals her signal if and only if $b_x \in B$. Throughout this section, we assume Assumption 3.3 holds (households have log preferences).

If the signal is not released, the payoff to a type γ bank is

$$\max\left\{\frac{p}{g}[1 - Rx], 0\right\} = \max\left\{\frac{p}{g}[1 - b_x], 0\right\}$$

and to a type δ bank:

$$\max\left\{\frac{1-p}{d}[1 - b_x], 0\right\}$$

Note that banks only get payoffs when there is full entry in the bond market, which simplifies these expressions.

The *relative gain* to being a type γ is

$$\Delta_0(g, b_x) := \lambda(g) \max\{1 - b_x, 0\}$$

where we define $\lambda(g) = w'(g) = \dfrac{p}{g} - \dfrac{1-p}{1-g}$. Both $\lambda(g)$ and $\Delta_0(g, b_x)$ are decreasing in g and equal zero at $g^* := p$. (Δ_0 is obviously zero everywhere if $b_x > 1$).

If the signal is released, type γ banks get interest rate $R_g = g/p$ and payoff $\dfrac{p}{g} - x$. Type δ banks may be rationed. They face interest rate

$$R_d = \phi(d + b)/(1 - p)$$

and get payoff

$$\max\left\{\frac{1-p}{d} - x\frac{d+b}{d}, 0\right\}$$

The relative gain to being a type γ is

$$\Delta_1(g, b_x) := \frac{p}{g} - x - \max\left\{\frac{1-p}{1-g} - x\frac{1-g+b}{1-g}, 0\right\} = \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, \frac{p}{g} - x\right\}$$

Ex ante, taking into account that the signal will be released in states $b_x \in B$, the expected gain from being a type γ is

$$H(g, B) := \int_\Omega \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x$$

In an equilibrium with $g \in (0, 1)$, banks must be indifferent between choosing to be type γ or type δ at date 0. Consequently, we must have $H(g, B) = 0$. Assuming this equation has a unique solution for g, then by choosing her information policy B, the regulator implicitly determines the fraction of type γ banks g. The following Lemma states that with log preferences, for any B, there is a unique, interior equilibrium.

Lemma 4.1. *(Uniqueness.) The constraint*

$$H(g, B) := \int_\Omega \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x = 0$$

uniquely defines g as a function of B, $g(B)$.

Proof. H is continuous and decreasing in g with $H(0, B) = \infty$, $H(1, B) = -\infty$. □

4.2 The Ex-Ante Optimal Disclosure Policy

In this section, we describe the optimal disclosure of information under commitment. Again, the regulator's problem is to choose a set $B \subset \Omega$ in which to release the signal, which in turn uniquely determines the fraction of γ banks $g \in (0,1)$. We can equivalently let the regulator choose both g and B, subject to the banks' indifference condition. Formally, the regulator's problem is

$$\begin{aligned}\max_{g,B} \int_{\Omega} U_0(g,b_x) f(b_x)\,\mathrm{d}b_x + \int_B (U_1(g,b_x) - U_0(g,b_x)) f(b_x)\,\mathrm{d}b_x \\ \text{s.t.} \int_{\Omega} \Delta_0(g,b_x) f(b_x)\,\mathrm{d}b_x + \int_B (\Delta_1(g,b_x) - \Delta_0(g,b_x)) f(b_x)\,\mathrm{d}b_x = 0\end{aligned}$$

where

$$\begin{aligned}U_0(g,b_x) &= w(g) - \min\{b_x, 1 + \ln b_x\} \\ U_1(g,b_x) &= w(g) - \min\{b_x, 1 - p + xg + (1-p)\ln\left(\frac{b_x - xg}{1-p}\right)\} \\ \Delta_0(g,b_x) &= \lambda(g)\max\{1 - b_x, 0\} \\ \Delta_1(g,b_x) &= \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, \frac{p}{g} - x\right\} \\ w(g) &= p\ln g + (1-p)\ln(1-g),\ \lambda(g) = w'(g)\end{aligned}$$

and f has support $\Omega \subseteq [x, \infty)$.

The following Proposition characterizes the optimal disclosure policy.

Proposition 4.2. *Define $\hat{b}_x(g) = xg/p, \check{b}_x(g) = xg + 1 - p$. The optimal strategy $\{g^{**}, B^{**}\}$ satisfies $g^{**} > g^*$, and for some $\bar{b}_x \in \Omega$,*

$$B^{**} = [x, \hat{b}_x(g^{**})] \cup [\bar{b}_x, \infty)$$

4.3 Interpretation

Before turning to the proof of Proposition 4.2, we describe the qualitative properties of the optimal policy. As a benchmark, it is useful to consider the unconstrained program in which the regulator ignores the constraint $H(g,B) = 0$, and can directly choose the level of diversification g. In this relaxed program, it would be optimal to set $g = g^*$, $B = \Omega$. Banks' investment is efficiently diversified between the two projects, because households are risk averse and value diversification. The regulator's private signal is always revealed, because this always increases social surplus.

The constraint $H(g,B) = 0$ means that this allocation is not implementable. Banks' entry decisions induce a tradeoff between efficient information revelation and diversification. Releasing information generally harms type δ banks, because they are misclassified as bad, and lose access to bond markets. Banks anticipate this ex ante, and will be less willing to choose δ projects. In equilibrium, the fraction of δ projects must fall until these projects are so scarce, and households are so poor in state D, that the expected benefit of investing in these projects compensates for the stigma of being misclassified as a bad bank.

Given this tradeoff, might it be better to reveal no information at all, and set $B = \emptyset$? No, because the benefit from information revelation is first-order, while the cost of deviating from efficient diversification is second order.

Given that it is optimal to disclose information sometimes, when should it be disclosed? The proposition states that disclosure is optimal both when b_x (the normalized fraction of bad banks) is below some threshold, and when b_x is above a threshold. In this sense, the optimal disclosure strategy is nonmonotonic. Take the upper threshold first. The intuition for this result is that the benefits of disclosure are increasing in b_x: when there are more bad banks, adverse selection is more severe, and the benefits from reducing adverse selection are larger. But the costs of disclosure are (weakly) decreasing in b_x. In the absence of disclosure, type δ banks would earn higher profits than type γ banks, because since $g > g^*$ in equilibrium, δ banks are relatively scarce. Releasing information removes this scarcity rent, discouraging δ-investment. But when b_x is high, these scarcity rents are small in any case, because *both* γ and δ banks pay higher interest rates. Thus the cost of releasing information is smaller. Putting these results together, the net benefit from disclosure is clearly increasing in b_x.

Next, consider the lower cutoff. Why can it be optimal to reveal the signal for low values of b_x? Take the extreme case where there are no bad banks, $b_x = x$. In this case, by revealing her signal, the regulator perfectly distinguishes γ and δ banks. As in Figure 2a above, this increases the interest rate charged to γ banks, and decreases the rate charged to δ banks. Intuitively, δ banks are valued more highly than γ banks, because they are relatively scarce. Distinguishing these banks, if possible, increases their profits, and encourages investment in δ projects ex ante. This is desirable, because $g > g^*$ in equilibrium, so the regulator would like to use all means to keep g as low as possible.

If b_x is only slightly larger than x, the same argument goes through - by Bayes' rule, the signal that a bank is *not* type γ still implies that the bank is extremely likely to be type δ, so the average 'not γ' bank is still more valuable than a γ bank. As b_x gets larger, eventually this is no longer true, and a 'not γ' bank is less valuable than a γ bank in expectation. At this point it is optimal to stop releasing the signal.

4.4 Proof

Next, we prove Proposition 4.2.

Lemma 4.3. *(Properties of Δ.). When $g > g*$:*

1. $\Delta_1(g, b_x) - \Delta_0(g, b_x) < 0$ *for* $b_x < \hat{b}_x(g)$. $\Delta_1(g, b_x) - \Delta_0(g, b_x) > 0$ *for* $b_x > \hat{b}_x(g)$.

2. $\Delta_1(g, b_x) - \Delta_0(g, b_x)$ *is increasing in* b_x *for* $b_x < \check{b}_x(g)$, *decreasing for* $b_x \in [\check{b}_x(g), 1]$, *and constant for* $b_x > 1$.

Proof. By calculation. □

Lemma 4.4. *(Feasible g.) In any feasible allocation,* $g \geq g^*$. *If* $g = g^*$, *then B has measure zero.*

Proof. Suppose by contradiction that $g < g^*$. Then $\Delta_0(g, b_x)$ is clearly nonnegative, and positive for some b_x. $\Delta_1(g, b_x) - \Delta_0(g, b_x)$ is increasing in b_x, and is positive for all $b_x > x$. So we must have

$$\int \Delta_0(g, b_x) f(b_x)\, \mathrm{d}b_x + \int_B [\Delta_1(g, b_x) - \Delta_0(g, b_x)] f(b_x)\, \mathrm{d}b_x > 0$$

for any B.

$\Delta_0(g^*, b_x) = 0$, $\Delta_1(g^*, b_x) = \min\{(b_x - x)/(1 - g^*), 1 - x\} > 0$ with probability 1, since $b_x > x$. So if (g^*, B) is feasible, B has measure zero. □

Lemma 4.5. *(Optimal g.)* $g^{**} > g^*$ *for any optimal policy* $\{g^{**}, B^{**}\}$.

Proof. Suppose by contradiction that $g = g^*$ is optimal: then by Lemma 4.4, B has measure zero. Consider the following class of deviations, indexed by $\varepsilon > 0$: Set $B = [1, 1 + \varepsilon)$. $g(\varepsilon)$ is defined by

$$\lambda(g) \int_x^1 (1 - b_x)\, \mathrm{d}F(b_x) + \left[\frac{p}{g} - x\right] \int_1^{1+\varepsilon} \mathrm{d}F(b_x) = 0$$

as a continuous, differentiable function of ε. The utility from such a deviation is then

$$V(\varepsilon) = w(g(\varepsilon)) - \int_x^\infty \min\{b_x, 1 + \ln b_x\}\, \mathrm{d}F(b_x) + \int_1^{1+\varepsilon} [U_1(g(\varepsilon), b_x) - U_0(g(\varepsilon), b_x)]\, \mathrm{d}F(b_x)$$

Taking derivatives and setting $\varepsilon = 0$, we have

$$V'(0) = w'(g(0))g'(0) + f(1)[U_1(g(0), 1) - U_0(g(0), 1)] = f(1)[U_1(g^*, 1) - U_0(g^*, 1)] > 0$$

where we use the facts that $g(0) = g^*$ and $w'(g^*) = 0$. Since small deviations increase utility, $g = g^*$ cannot be optimal. □

Lemma 4.6. *(Low* b_x*.) The optimal strategy* $\{g^{**}, B^{**}\}$ *satisfies*

$$[x, \hat{b}_x(g^{**})] \subset B^{**}, (\hat{b}_x(g^{**}), \check{b}_x(g^{**})) \cap B^{**} = \emptyset.$$

Proof. Suppose to the contrary that there exists an interval $[b_x^1, b_x^2] \subset [x, \hat{b}_x(g)]$ but $[b_x^1, b_x^2] \not\subset B^{**}$. Consider the following class of deviations, indexed by $\varepsilon > 0$: Augment B^{**} with $[b_x^1, b_x^1 + \varepsilon)$, and define $g(\varepsilon)$ as a solution to

$$I(g, B) := H(g, B) + \int_{b_x^1}^{b_x^1 + \varepsilon} (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\, \mathrm{d}b_x = 0,$$

where $I(g, B)$ is differentiable with respect to ε. Since $I_g < 0$, $I_\varepsilon < 0$ (since $\Delta_1 < \Delta_0$ when $b_x < \hat{b}_x$), thus $g'(\varepsilon) < 0$.

Such a deviation increases utility to

$$V(\varepsilon) = \int_\Omega U_0(g(\varepsilon), b_x) f(b_x)\, \mathrm{d}b_x + \int_{B^{**}} (U_1(g(\varepsilon), b_x) - U_0(g(\varepsilon), b_x)) f(b_x)\, \mathrm{d}b_x$$

where we use the feature from Lemma 3.9 that $U_0 = U_1$ when $b_x < \check{b}_x(g)$. Since $g'(\varepsilon) < 0$ and U_0, U_1 are decreasing in g (by Lemma 3.9), this small deviation increases utility, so the original allocation cannot be optimal.

Similarly, one can show that B^{**} cannot intersect $(\hat{b}_x(g), \check{b}_x(g))$ because $\Delta_1 > \Delta_0$ for every $\beta \in (\hat{b}_x(g), \check{b}_x(g))$. □

Lemma 4.7. *(High b_x.) $B^{**} \setminus [x, \check{b}_x(g))$ is an interval $[\bar{b}_x, \infty)$ for some $\bar{b}_x \geq \check{b}_x(g)$.*

Proof. First, we show that B^{**} must contain some $b_x \geq \check{b}_x(g)$. Suppose not; then by Lemma 4.6, $B^{**} = [x, \hat{b}_x(g)]$ for some $g > g^*$, and the associated expected welfare is $\int_\Omega U_0(g, b_x)\, \mathrm{d}F(b_x)$ since $U_0 = U_1$ for $b_x < \hat{b}_x(g)$. Consider the following deviation: set $B = \emptyset$ and $g = g^*$. Since U_0 is decreasing in g by Lemma 3.9, clearly this deviation strictly increases utility. Therefore, the original disclosure rule $[x, \hat{b}_x(g)]$ cannot be optimal.

Next, we show that B^{**} must be 'monotonic' above $\check{b}_x(g)$. Suppose to the contrary that there exist two sets $B_1, B_2 \subset [\check{b}_x(g), \infty)$ such that $\mathbb{P}(b_x \in B_1), \mathbb{P}(b_x \in B_2) \neq 0$, $B_1, B_2 \subset B$, and $b_x^1 < b_x^2$ for all $b_x^1 \in B_1, b_x^2 \in B_2$. Let $\overline{b_{x1}} := \max\{b_x | b_x \in B_1\}$ and $\underline{b_{x_2}} := \min\{b_x | b_x \in B_2\}$. For a given g, consider

$$H(B^{**}) = \int_{B^{**}} [\Delta_1(g, b_x) - \Delta_0(g, b_x)] f(b_x)\, \mathrm{d}b_x =: \int_{B^{**}} h(b_x)\, \mathrm{d}b_x$$

Find nonempty subsets $B_1' \subset B_1$ and $B_2' \subset (\overline{b_{x1}}, \underline{b_{x_2}})$ such that $\mathbb{P}(b_x \in B_1'), \mathbb{P}(b_x \in B_2') \neq 0$ and $H(B_1') = H(B_2') \neq 0$. Consider the following deviation: augment B with B_2', but remove B_1'. By construction, this keeps the value of the constraint unchanged, so the same value of g remains feasible.

Define

$$k(b_x) = \frac{U_1(g, b_x) - U_0(g, b_x)}{\Delta_1(g, b_x) - \Delta_0(g, b_x)}$$

$k(b_x)$ is positive and increasing for $b_x > \check{b}_x(g)$ by Lemmas 3.9 and 4.3. The expected welfare of this new disclosure rule is

$$
\begin{aligned}
&\int_{B_2'} [U_1(g,b_x) - U_0(g,b_x)] f(b_x)\,\mathrm{d}b_x - \int_{B_1'} [U_1(g,b_x) - U_0(g,b_x)] f(b_x)\,\mathrm{d}b_x \\
&= \int_{B_2'} k(b_x) h(b_x)\,\mathrm{d}b_x - \int_{B_1'} k(b_x) h(b_x)\,\mathrm{d}b_x \\
&> \min_{b_x \in B_2'} \{k(b_x)\} \cdot \int_{B_2'} h(b_x)\,\mathrm{d}b_x - \max_{b_x \in B_1'} \{k(b_x)\} \cdot \int_{B_1'} h(b_x)\,\mathrm{d}b_x \\
&\geq \min_{b_x \in B_2'} \{k(b_x)\} \cdot \left(\int_{B_2'} h(b_x)\,\mathrm{d}b_x - \int_{B_1'} h(b_x)\,\mathrm{d}b_x \right) = 0,
\end{aligned}
$$

which shows that this deviation increases the expected social welfare while it does not affect the banks' incentive for portfolio choice. We can thus conclude the original disclosure rule is not optimal. □

4.5 More general preferences

Disclosing the regulator's information is costly because households are risk averse. It is therefore of interest to describe how the optimal policy changes as households' risk aversion changes. To do this, we need to solve for optimal policy with more general preferences. This introduces additional complications.

When households are less risk-averse than log, multiple equilibria are possible, as is standard in adverse selection models with risk-neutral agents. If many δ-banks enter the market, the average 'not γ' bank is probably type δ, and is valuable, thus investors will buy its bonds at date 1; this confirms that it was a good idea for these banks to invest. If few such banks enter, the average 'not γ' bank is probably a bad bank, and is not valuable, thus investors will not buy its bonds, confirming that this it was a good idea for banks *not* to choose project δ. Mathematically, adverse selection tends to make the gain to being a type γ bank *increasing* in g. On the other hand, risk aversion makes the return decreasing in g: when δ banks are scarce, they are valued more highly by the market. It so happens that with log utility, risk aversion always outweighs adverse selection, and there is a unique equilibrium. When households are less risk averse than log, however, multiple equilibria are possible.

In this section, we prove nonetheless that our main result goes through, with the following modification. If we allow the regulator to choose B and to select g among the equilibria consistent with B, optimal policies have the form described above. In particular, the regulator always wants to choose the equilibrium with the lowest g (closest to g^*). However, such policies now involve the risk that the economy may coordinate on the bad equilibrium with less diversification.

Let $u(c) = \dfrac{c^{1-\sigma}}{1-\sigma}$, $\sigma \in (0, \infty)$. Define $w_G(g) = pg^{1-\sigma}$, $w_D(g) = (1-p)(1-g)^{1-\sigma}$, $w(g) = w_G(g) + w_D(g)$.

Without the signal, there is partial entry if $b_x > w(g)$. The interest rate satisfies $xR =$

$\min\left\{\frac{b_x}{w(g)}, 1\right\}$. With the signal, there is partial entry if $b_x > \check{b}_x(g) := w_D(g) + xg$. The interest rate satisfies $xR_D = \frac{b_x - xg}{w_D(g)}$. The profit of G banks in this case is $pg^{-\sigma} - x$. The profit of D banks is

$$(1-p)(1-g)^{-\sigma} - \frac{b_x - xg}{1-g}$$

Also define $\lambda(g) = pg^{-\sigma} - (1-p)(1-g)^{-\sigma}$.

$$\begin{aligned}
U_0(g, b_x) &= \max\left\{\frac{w(g)}{1-\sigma} - b_x, \frac{\sigma b_x^{1-\frac{1}{\sigma}} w(g)^{\frac{1}{\sigma}}}{1-\sigma}\right\} \\
U_1(g, b_x) &= \max\left\{\frac{w(g)}{1-\sigma} - b_x, \frac{w_G(g)}{1-\sigma} - xg + \frac{\sigma[b_x - xg]^{1-\frac{1}{\sigma}} w_D(g)^{\frac{1}{\sigma}}}{1-\sigma}\right\} \\
\Delta_0(g, b_x) &= \lambda(g) \max\left\{1 - \frac{b_x}{w(g)}, 0\right\} \\
\Delta_1(g, b_x) &= \min\left\{\lambda(g) + \frac{b_x - x}{1-g}, pg^{-\sigma} - x\right\}
\end{aligned}$$

Again, the regulator's problem is

$$\begin{aligned}
&\max_{g,B} \int_\Omega U_0(g, b_x) f(b_x)\,\mathrm{d}b_x + \int_B (U_1(g, b_x) - U_0(g, b_x)) f(b_x)\,\mathrm{d}b_x \\
&\text{s.t.} \int_\Omega \Delta_0(g, b_x) f(b_x)\,\mathrm{d}b_x + \int_B (\Delta_1(g, b_x) - \Delta_0(g, b_x)) f(b_x)\,\mathrm{d}b_x = 0
\end{aligned}$$

Note that by allowing the regulator to choose any g, B consistent with the entry condition, we essentially assume away the multiple equilibria issue, allowing the regulator to implement whichever g she prefers consistent with the same B.

With this caveat, the following Proposition characterizes the optimal disclosure policy, showing that our result for the log utility case can be generalized to CRRA utilities.

Proposition 4.8. *For general CRRA perferences, the optimal allocation rule $\{g^{**}, B^{**}\}$ satisfies (i) $g^{**} > g^*$; and (ii) $B^{**} = [x, \hat{b}_x(g^{**})] \cup [\bar{b}_x, \infty)$ for some $\bar{b}_x \in \Omega$.*

Proof. In the Appendix. □

Figure 3 illustrates numerically how the optimal policy depends on households' risk aversion. The measure of bad banks, b, is uniformly distributed over $[0, 10]$. We set the cost of investing, x, equal to 0.2, and the probability that state G realizes, p, equal to 0.5. Figure 3 plots the optimal value of g, its first best value g^*, and the optimal lower and upper bounds, $F(\hat{b}_x(g))$ and $F(\bar{b}_x)$, as functions of σ, households' risk aversion. The lower and upper bounds are normalized to represent probabilities, so the area in between the blue and black lines represents the probability that the regulator does not reveal her signal.

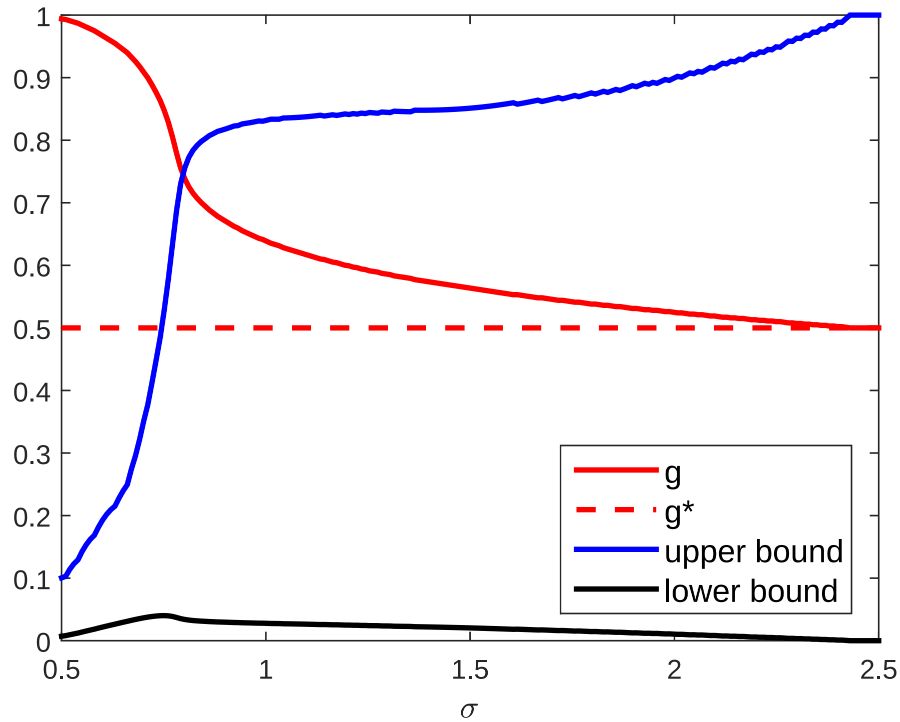


Figure 3 The optimal policy as a correspondence of σ

When households are not too risk averse, it is optimal to reveal the signal in most states of the world. This involves a large gap between the efficient level of investment in γ banks, $g^* = 0.5$, and the actual level, which is close to 1. But because households are not too risk-averse, they are willing to under-diversify in return for the efficiency gains associated with revealing the signal in bad states of the world and reducing inefficient investment. Moving left to right, as households become more risk averse, it is optimal to reveal the signal less often in order to induce a lower level of g, closer to the first-best level of $g^* = 0.5$.

5 Conclusion

Stress tests have moved from an exceptional measure of crisis management to a routine part of financial regulation. In a crisis, stress tests can reduce bank opacity and prevent market shutdowns. But as a routine policy, stress tests may encourage banks to mimic regulators' models, pass the tests, and ignore their own measures of risk. This reduces diversification, leaving the financial system vulnerable to the risk that regulators' models turn out to be wrong. We presented a simple model to understand this tradeoff, and showed that the optimal policy is to release stress test results only in severe crises.

References

Bernanke, Ben. 2013. "Stress Testing Banks: What Have We Learned?" Remarks by Chairman Ben S. Bernanke at the 'Maintaining Financial Stability: Holding a Tiger by the Tail' financial markets conference sponsored by the Federal Reserve Bank of Atlanta, Stone Mountain, Georgia.

Bond, Philip, and Itay Goldstein. 2015. "Government intervention and information aggregation by prices." *The Journal of Finance*, 70(6): 2777–2812.

Bouvard, Matthieu, Pierre Chaigneau, and Adolfo De Motta. 2015. "Transparency in the Financial System: Rollover Risk and Crises." *The Journal of Finance*, 70(4): 1805–1837.

Faria-e Castro, Miguel, Joseba Martinez, and Thomas Philippon. 2015. "Runs versus Lemons: Information Disclosure and Fiscal Capacity." National Bureau of Economic Research Working Paper 21201.

Flannery, Mark, Beverly Hirtle, and Anna Kovner. 2015. "Evaluating the Information in the Federal Reserve Stress Tests." Federal Reserve Bank of New York Staff Reports 744.

Frame, W. Scott, Kristopher S. Gerardi, and Paul S. Willen. 2015. "The Failure of Supervisory Stress Testing: Fannie Mae, Freddie Mac, and OFHEO." Federal Reserve Bank of Boston Working Papers 15-4.

Gigler, Frank, Chandra Kanodia, Haresh Sapra, and Raghu Venugopalan. 2014. "How Frequent Financial Reporting Can Cause Managerial Short-Termism: An Analysis of the Costs and Benefits of Increasing Reporting Frequency." *Journal of Accounting Research*, 52(2): 357–387.

Glasserman, Paul, and Gowtham Tangirala. 2015. "Are the Federal Reserve's Stress Test Results Predictable?" Office of Financial Research, US Department of the Treasury Working Papers 15-02.

Goldstein, Itay, and Haresh Sapra. 2013. "Should Banks' Stress Test Results be Disclosed? An Analysis of the Costs and Benefits." *Foundations and Trends in Finance*, 8(1): 1–54.

Goldstein, Itay, and Yaron Leitner. 2015. "Stress Tests and Information Disclosure." Federal Reserve Bank of Philadelphia Working Paper.

Hirshleifer, Jack. 1971. "The Private and Social Value of Information and the Reward to Inventive Activity." *American Economic Review*, 61(4): 561–74.

Leitner, Yaron. 2014. "Should regulators reveal information about banks?" *Business Review*, , (Q3): 1–8.

Morgan, Donald P., Stavros Peristiani, and Vanessa Savino. 2014. "The Information Value of the Stress Test." *Journal of Money, Credit and Banking*, 46(7): 1479–1500.

Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information." *American Economic Review*, 92(5): 1521–1534.

Spargoli, Fabrizio. 2013. "Bank Recapitalization and the Information Value of a Stress Test in a Crisis." Universitat Pompeu Fabra Working Paper.

Tett, Gillian. 2015. "Stress Tests for Banks Are a Predictable Act of Public Theatre." The Financial Times.