Qualifying Exam—Practice Problems

1. A consumer has preferences over two goods, x_1 and x_2, represented by

$$u(x_1, x_2) = \ln x_1 + x_2.$$

Let the price of x_1 be p and normalize the price of x_2 by 1. Let also denote the consumer's income by m.

(a) Derive the Marshallian demands for x_1 and x_2.

(b) Derive the expenditure function.

(c) Suppose that the price of x_1 raises from p^0 to $p^1 > p^0$. Derive the consumer surplus (CS), the equivalent variation (EV), and the compensating variation (CV). Show that they are the same with each other. Explain why it is.

2. A consumer has a Bernoulli utility function $u(w) = \ln w$. He is offered the opportunity to bet on the flip of a coin that has a probability 1/2 of coming up heads. If he bets \$$r$, he will have $w + r$ if head comes up and $w - r$ if tail comes up. Solve for the optimal r.

3. There is a firm that produces output y with the following production technology:

$$f(x_1, x_2) = x_1 + x_2.$$

Let the price of x_i be w_i for $i = 1, 2$.

(a) Derive the cost function $c(\mathbf{w}, y)$

(b) Derive the conditional factor demand function $x(\mathbf{w}, y)$.

4. Consider a two-consumer, two-good exchange economy. Utility functions and endowments are

$$u^1(x_1, x_2) = (x_1 x_2)^2 \quad \text{and} \quad e^1 = (18, 4),$$
$$u^2(x_1, x_2) = \ln(x_1) + \ln(x_2) \quad \text{and} \quad e^2 = (3, 6).$$

(a) Characterize the set of Pareto-efficient allocations.

(b) Find a Walrasian equilibrium and compute the equilibrium allocations.